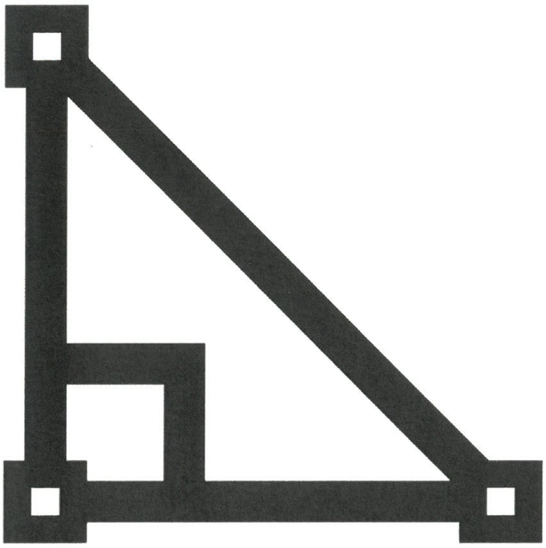
Mathematics
Transition to Year 11

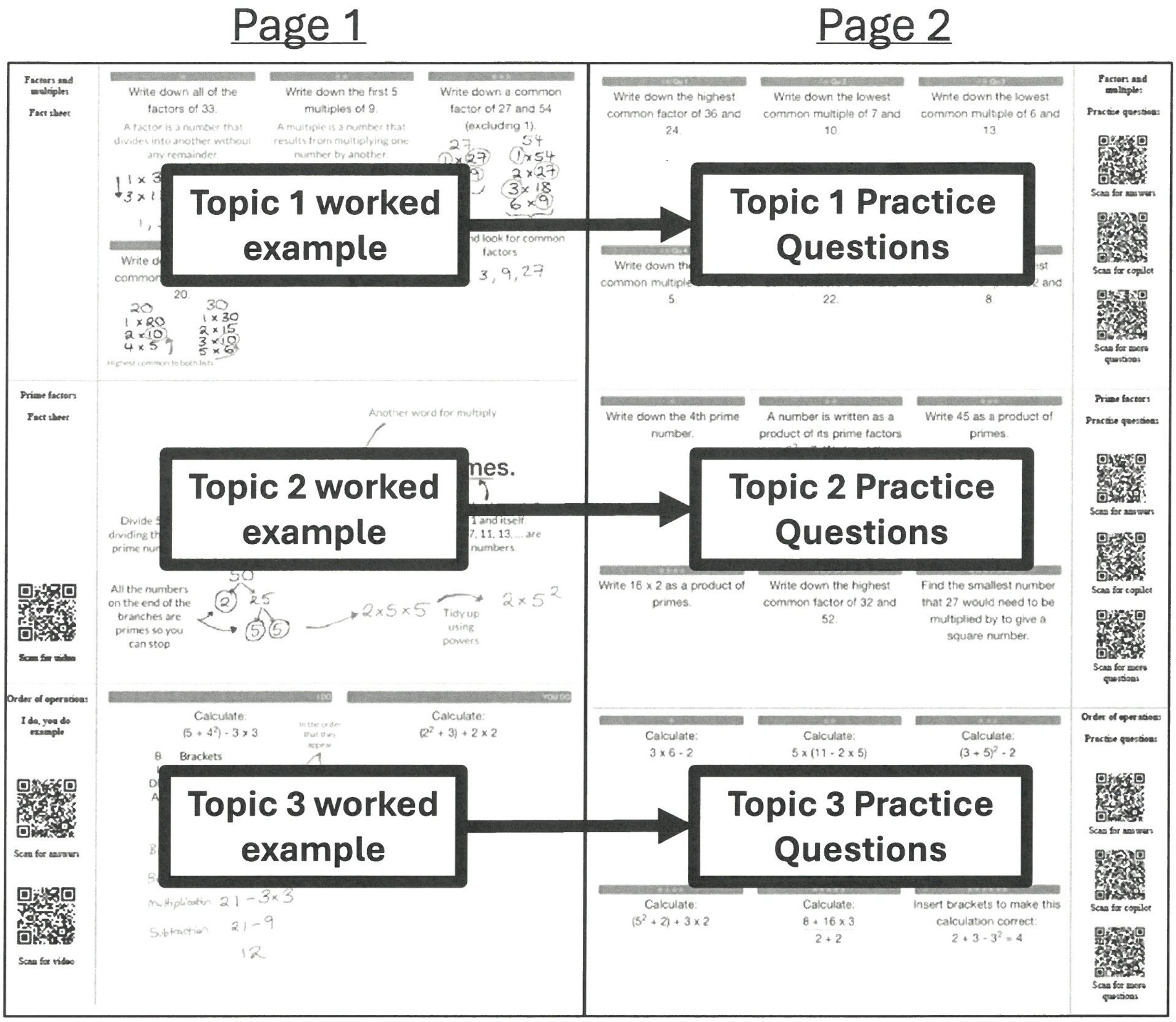
Higher Plus



Name: ______________________________

How to use this booklet

Work across each row of a double page.



Remember to use the solutions to mark your work.

Algebraic fractions

I do, you do example

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I DO

Simplify:

$$\frac{d^2}{d^2 + 14d + 48} \times \frac{d + 6}{d^3}$$

With algebraic fractions, you follow the normal techniques for operating with numeric fractions, but also use factorising to simplify where possible.

$$\frac{d^2}{(d^2 + 14d + 48)} \times \frac{d+6}{d^3}$$

Factorise

$$\equiv \frac{d^2}{(d+6)(d+8)} \times \frac{d+6}{d^3}$$

Cancel common factors

$$\equiv \frac{\cancel{d^2}\cancel{(d+6)}}{\cancel{(d+6)}(d+8)d^{\cancel{3}1}} \equiv \frac{1}{d(d+8)}$$

YOU DO

Simplify:

$$\frac{a^2 + 15a + 56}{a^2 + 5a - 14} \times \frac{a - 2}{a + 7}$$

Change the subject

I do, you do example

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I DO

Make c the subject:

cb - a = e

To make c the subject we use inverse operations to isolate c on one side of the equals sign.

$$cb - a = e$$
$$+a \qquad +a$$

$$cb = e + a$$
$$\div b \qquad \div b$$

$$c = \frac{e+a}{b}$$

When something isn't divisible, leave the answer as a fraction

YOU DO

Make e the subject:

ef + b = a

Find the equation of a perpendicular line

I do, you do example

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I DO

Write down the equation of a line perpendicular to y=-3x+5 that passes through the point (3,7). Give your answer in the form ax+by=c.

A perpendicular line has a gradient that is the negative reciprocal of the original gradient. When multiplied together, the product of the gradients is -1.

$m = -3$ Gradient of original line

$-\frac{1}{m} = \frac{1}{3}$ Gradient of perpendicular line

We know the line passes through (3,7) so if $y = \frac{1}{3}x + c$, then:

$$7 = \frac{1}{3}(3) + c$$
$7 = 1 + c$ therefore c = 6

$$y = \frac{1}{3}x + 6 \Rightarrow 3y = x + 18 \Rightarrow \overset{ax}{-x} + \overset{by}{3y} = \overset{c}{18}$$

YOU DO

Write down the equation of a line perpendicular to y=8-6x that passes through the point (6,-8). Give your answer in the form ax+by=c.

Algebraic fractions

Practise questions

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★

Simplify:

$$\frac{48a^7g^4}{6a^5g^7}$$

★★

Calculate:

$$\frac{a}{d} \div \frac{e}{h}$$

★★★

Simplify:

$$\frac{g^2 - 2g - 15}{gf - 5f}$$

★★★★

Simplify:

$$\frac{7c}{c - 3} \times \frac{c}{7}$$

★★★★★

Simplify:

$$\frac{3}{c + 7} - \frac{6}{2c + 3}$$

★★★★★★

Solve:

$$\frac{12}{f + 17} - \frac{9}{2f + 13} = 3$$

Change the subject

Practise questions

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★

Make c the subject:

$$c + e = b$$

★★

Make d the subject:

$$dc - f = b$$

★★★

Make c the subject:

$$\frac{cb - a}{e} = d$$

★★★★

Make a the subject:

$$\frac{\sqrt{a} - c}{b} = e$$

★★★★★

Make r the subject:

$$A = \pi r^2$$

★★★★★★

Make c the subject:

$$e = ac + 2c$$

Find the equation of a perpendicular line

Practise questions

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★

The gradient of a line is $\frac{4}{7}$ What is the gradient of a perpendicular line?

★★

Line J passes through (-9,-14) and (-14,-11). Line Q passes through (2,-8) and (3,11). Is line J perpendicular to line Q?

★★★

Write down the equation of a line perpendicular to $y = 5x - 6$.

★★★★

Prove that the following two lines are perpendicular:

$$y = 2x + 5$$
$$y = -{}^{1}/_{2}x + 8$$

★★★★★

The equations of 2 lines are given below:

$$y = 7x + 3$$
$$y = 8 + {}^{1}/_{7}x$$

Are they perpendicular?

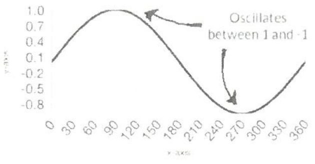
★★★★★★

Write down the equation of a line perpendicular to $y=-5x+5$ that passes through the point (-5,-2). Give your answer in the form ax+by=c.

Trigonometry graphs

Fact sheet

What is the equation of the following graph?



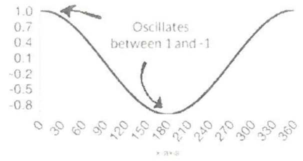
Oscillates between 1 and -1

This is the sine graph:

$y = \sin x°$

$x°$	y
0	0
30	0.5
90	1
150	0.5
180	0

What is the equation of the following graph?



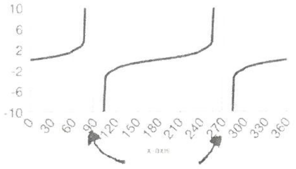
Oscillates between 1 and -1

This is the cosine graph:

$y = \cos x°$

$x°$	y
0	1
60	0.5
90	0
120	-0.5
180	-1

What is the equation of the following graph?



Asymptotes at 90 and 270 (the lines tends to infinity)

This is the tangent graph:

$y = \tan x°$

$x°$	y
0	0
45	1
90	∞
135	-1
180	0

Complete the square

I do, you do example

Scan for answers

Scan for video

I DO

Write the following expression in completed square form:

$x^2 + 4x - 1$

The completed square form is:

$$(x + d)^2 + e$$

$\frac{1}{2}b$ ↗ ↖ $c - d^2$

So for a quadratic above that is written in the form $ax^2 + bx + c$:

a = 1, b = 4 and c = -1

$d = \frac{1}{2}(4) = 2$

$e = -1 - (2)^2 = -1 - 4 = -5$

$(x + 2)^2 - 5$

From this form, we can also get the turning point of the quadratic: (-2, -5)

YOU DO

Write the following expression in completed square form:

$x^2 - 6x - 2$

Solve a quadratic by completing the square

I do, you do example

Scan for answers

Scan for video

I DO

Solve the following quadratic by completing the square (leave in exact form):

$x^2 + 2x - 2 = 0$

Complete the square

$(x + 1)^2 - 2 - (1)^2 = 0$

$(x + 1)^2 - 2 - 1 = 0$

$(x + 1)^2 - 3 = 0$

Begin to rearrange to make x the subject

$(x + 1)^2 = 3$

$x + 1 = \pm\sqrt{3}$

$x = -1 \pm \sqrt{3}$

There are 2 answers in this case due to the + and - of the root of 3

YOU DO

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 4x - 4 = 0$

Trigonometry graphs

Practise questions

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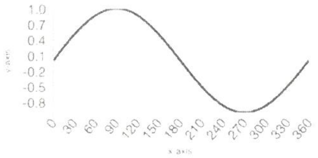
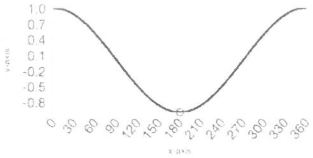
★

What is the equation of the following graph?



★★

State the coordinates of the point on the graph:

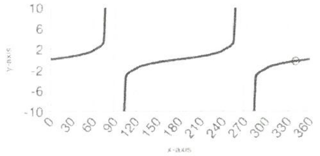


★★★

Sketch the graph of y = cos$x$ for $0 \leq x \leq 360$.

★★★★

When $x = 340°$, tan$x$ ≈ -0.4. Find another value of x that satisfies tan$x$ ≈ -0.4.



★★★★★

When $x = 225°$, sin$x$ ≈ -0.7. Find another value of x that satisfies sin$x$ ≈ -0.7.

Complete the square

Practise questions

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★

Find the value of a in the following:

$x^2 + 8x + 19 = (x + 4)^2 + a$

★★

Find the value of a in the following:

$x^2 - 18x + 88 = (x - 9)^2 + a$

★★★

Write the following expression in completed square form:

$x^2 - 2x - 7$

★★★★

Write the following expression in completed square form:

$x^2 + 7x + 7$

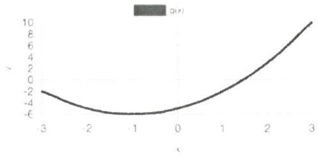
★★★★★

Find the minimum point of the following function:

$f(x) = x^2 - 6x - 3$

★★★★★★

$f(x) = x^2$ has translated to $g(x)$. Identify $g(x)$.



Solve a quadratic by completing the square

Practise questions

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★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 + 6x - 8 = 0$

$(x + 3)^2 + a = 0$

★★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 4x - 1 = 0$

$(x - 2)^2 + a = 0$

★★★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 2x - 4 = 0$

★★★★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 3x - 5 = 0$

★★★★★

Solve the following quadratic by completing the square (to 2dp):

$5x^2 - 5x - 3 = 0$

★★★★★★

Solve the following quadratic by completing the square (to 2dp):

$8x^2 + 17x - 18 = 0$

Solve a quadratic by using the formula

I do, you do example

Scan for answers

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I DO

Using the quadratic formula, solve the following equation (to 2 d.p.):

$5x^2 - 13x + 3 = 0.$

a b c

A quadratic can be solved by using the quadratic formula when it is written in the form $ax^2 + bx + c = 0$ and the discriminant $b^2 - 4ac \geq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4 \times 5 \times 3}}{2 \times 5}$$

Simplify

$$x = \frac{13 \pm \sqrt{169 - 60}}{10} = \frac{13 \pm \sqrt{109}}{10}$$

Evaluate

$$x = \frac{13 + \sqrt{109}}{10} \approx 2.34 \qquad x = \frac{13 - \sqrt{109}}{10} \approx 0.26$$

YOU DO

Using the quadratic formula, solve the following equation (to 2 d.p.):

$10x^2 - 3x - 9 = 0.$

Solve quadratic inequalities

I do, you do example

Scan for answers

I DO

Solve:

$x^2 - 11x + 28 \leq 0$

Solve the quadratic inequality like a quadratic equation

Factorising gives:

$y \leq 0$

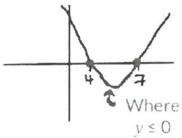
$$(x - 4)(x - 7) \leq 0$$

So when $x = 4$, $y = 0$

And when $x = 7$, $y = 0$

This gives the points of intersection with the x-axis

Sketch the inequality graph:



Where $y \leq 0$

So:
$4 \leq x \leq 7$

YOU DO

Solve:

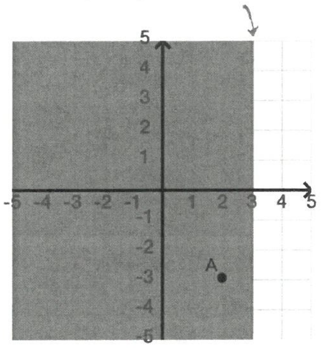
$x^2 - 8x + 12 \leq 0$

Graphical inequalities

Fact sheet

Scan for video

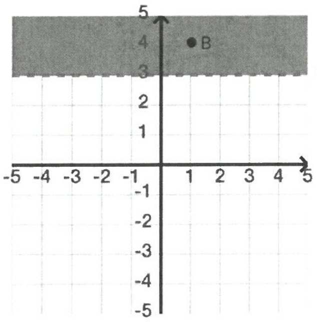
The line is $x = 3$. This means x could equal 3 and any value in the shaded region. A dotted line would mean x could not equal 3 but could equal any value inside the shaded region.



Point A in the shaded region has coordinates (2, -3). The x-coordinate of A is 2 which is less than 3. Any point in the shaded region will also have an x-coordinate less than 3. The shaded region can therefore be represented by:

$x \leq 3$ Due to the solid line

The line is $y = 3$ but because it is a dotted line y could not equal 3 but could equal any value inside the shaded region. A solid line would mean it could equal 3 and any value inside the shaded region.



Point B in the shaded region has coordinates (1, 4). The y-coordinate of B is 4 which is more than 3. Any point in the shaded region will also have a y-coordinate more than 3. The shaded region can therefore be represented by:

$y > 3$ Due to the dotted line

Solve a quadratic by using the formula

Practise questions

★

Substitute a = 2, b = 10 and c = 2 into the formula:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

★★

Substitute a = 9, b = -6 and c = -9 into the formula:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

★★★

Tia correctly used the quadratic formula to get:

$$x = \frac{10 \pm \sqrt{(100 + 468)}}{18}$$

Write down the quadratic equation Tia was solving.

★★★★

Use the disciminant to determine how many solutions the following quadratic will have:

$-13x^2 + 9x - 13 = 0$.

★★★★★

Using the quadratic formula, solve the following equation (to 2 d.p.):

$-7x^2 + 1x + 11 = 0$.

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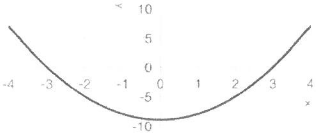
Solve quadratic inequalities

Practise questions

★

$f(x) = x^2 - 9$ is shown.

Solve $f(x) < 0$.



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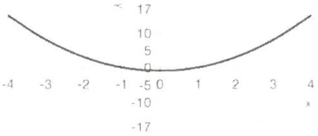
$f(x) = x^2 - 1$ is shown.

Solve $f(x) \geq 0$.



★★★

Solve:

$x^2 - 81 \leq 0$

★★★★

Solve:

$x^2 - 15x + 56 \leq 0$

★★★★★

Solve:

$3x^2 + 5x + 2 \leq 0$

★★★★★★

Solve:

$12x^2 \leq -31x - 20$

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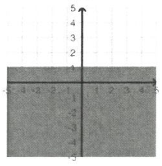
Graphical inequalities

Practise questions

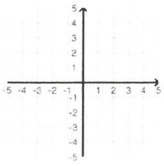
★

Identify the inequality that satisfies the given region:



★★

Clearly show the region that satisifies $x \leq 3$:



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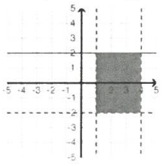
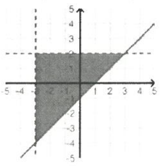
Identify the inequalities that satisfy the region:



★★★★

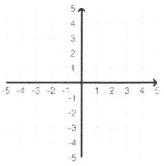
Identify the inequalities that satisfy the region:



★★★★★

Find all points that satisfy:

$x \geq -1 \quad y < 3 \quad y > x$



Given that x and y are integers.

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Understand quadratic sequences

I do, you do example

Scan for answers

Scan for video

I DO

Find the nth term for the following sequence:

8, 13, 20, 29, ...

5 7 9

2 2

Unlike linear sequences, quadratic sequences have a second line of common differences. The rule always comes in the form $ax^2 + bx + c$, where a is half the second line of differences.

$a = \frac{1}{2}(2) = 1$

The rule is n^2+bn+c.

Notice if you take n away, you get bn + c (a linear rule)

n^2+bn+c	8	13	20	29
n^2	1	4	9	16
$bn+c$	7	9	11	13

2n + 5

So the rule is n^2+2n+5

YOU DO

Find the nth term for the following sequence:

5, 10, 17, 26, ...

Solve non linear simultaneous equations

I do, you do example

Scan for answers

Scan for video

I DO

Solve for x and y:

① $x^2 + y^2 = 34$

② $x + y = 2$

Label the equations to help structure your work

First we need to rearrange (2) for either x or y:

③ $y = 2 - x$

Substitute equation (3) into equation (1):

Expand and simplify

$x^2 + (2-x)^2 = 34$

$x^2 + 4 - 4x + x^2 = 34$

$2x^2 - 4x - 30 = 0$

Make it equal to 0

Solve the quadratic using a suitable method:

Factorise

$2x^2 + 6x - 10x - 30 = 0$

$2x(x+3) - 10(x+3) = 0$

$(2x-10)(x+3) = 0$

$x = 5 \quad x = -3$

Substitute both x into equation (3) to find y:

$y = 2-5 = -3 \quad y = 2--3 = 5$

YOU DO

Solve for x and y:

$x^2 + y^2 = 37$

$x + y = 7$

Area/Volume scale factors

I do, you do example

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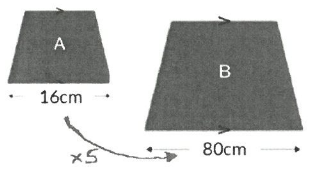
Scan for video

I DO

A and B are similar shapes. The area of A is 80cm^2. Find the area of B?



Similar shapes are 2 of the same shape but different in size.

Their lengths are linked by a scale factor.
Their areas are linked by (scale factor)^2.
Their volumes are linked by (scale factor)^3.

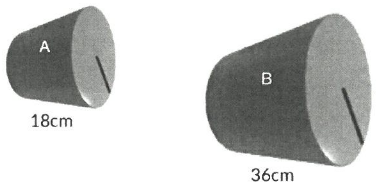
Scale factor $= \frac{80}{16} = 5$

Area scale factor $= 5^2 = 25$

So area of shape B $= 80 \times 25 = 2000\text{cm}^2$

YOU DO

A and B are similar. The volume of B is 720cm^3. Find the volume of A?



Understand quadratic sequences

Practise questions

★

Continue the sequence for the next 2 terms:
5, 15, 31, 53, ...

★★

Find the nth term rule for the following sequence:
6, 24, 54, 96, ...

★★★

Find the nth term for the following sequence:
10, 18, 28, 40, ...

★★★★

Find the nth term for the following sequence:
5, 12, 25, 44, ...

★★★★★

Find the nth term for the following sequence:
-12, -30, -58, -96, ...

★★★★★★

Find the 80th term in the following sequence:
-7, -19, -37, -61, ...

Scan for answers

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Scan for more questions

Solve non linear simultaneous equations

Practise questions

★

Solve for x and y:
$y = x^2$
$y = -7x - 12$

★★

Solve for x and y:
$y = x^2 + 3x - 24$
$y = 3x + 25$

★★★

Solve for x and y:
$xy = -12$
$y = x + 7$

★★★★

Solve for x and y:
$x^2 + y^2 = 17$
$x + y = 3$

★★★★★

Solve for x and y (to 2dp):
$x^2 + y^2 = 27$
$x + y = 6$

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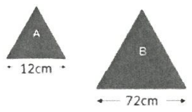
Area/Volume scale factors

Practise questions

★

A and B are similar shapes. The area of A is 36cm^2. Find the area of B?



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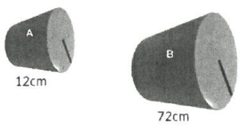
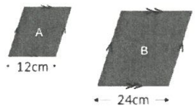
A and B are similar. The volume of A is 60cm^3. Find the volume of B?



★★★

A and B are similar shapes. The area of B is 192cm^2. Find the area of A?



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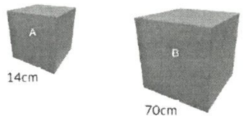
A and B are similar. The volume of B is 8750cm^3. Find the volume of A?



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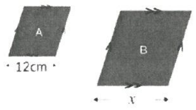
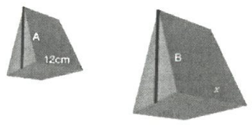
A and B are similar. The area of A is 48cm^2 and B is 3072cm^2. Find x?



★★★★★★

A and B are similar. The volume of A is 60cm^3 and B is 3840cm^3. Find x.



Scan for answers

Scan for copilot

Scan for more questions

Sine and cosine rules

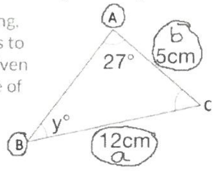
Fact sheet

Scan for video

Sine and cosine rules

Calculate y to 1dp.

When labelling, opposite sides to an angle are given the lowercase of the letter



The sine rule is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ Use for calculating sides

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$ Use for calculating angles

Use the sine rule where you have opposites (e.g. a and A, b and B):

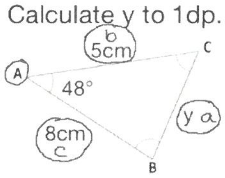
$$\frac{\sin 27}{12} = \frac{\sin B}{5}$$ Rearrange

$$\sin B = \frac{\sin 27}{12} \times 5$$

$$B = \sin^{-1}\left(\frac{\sin 27}{12} \times 5\right) \approx 10.9°$$

Sine and cosine rules

Calculate y to 1dp.



The cosine rule is:

$$a^2 = b^2 + c^2 - 2bc \times \cos A$$

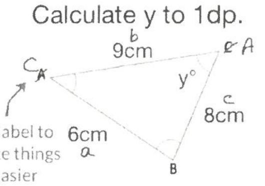
Always remember you can relabel the triangle if needed.

$$a^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 48$$
$$= 25 + 64 - 2 \times 5 \times 8 \times \cos 48$$
$$= 25 + 64 - 80\cos 48$$
$$= 89 - 80\cos 48$$
$$a = \sqrt{89 - 80\cos 48}$$
$$\approx 6.0\text{cm}$$

Always remember to square root to find a

Sine and cosine rules

Calculate y to 1dp.



Relabel to make things easier

Rearranging the cosine rule to find an angle we get:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{9^2 + 8^2 - 6^2}{2 \times 9 \times 8}$$
$$= \frac{81 + 64 - 36}{2 \times 9 \times 8}$$
$$= \frac{81 + 64 - 36}{144}$$
$$= \frac{109}{144}$$
$$A = \cos^{-1}\left(\frac{109}{144}\right) \approx 40.8°$$

Operate with surds

Fact sheet

★

Simplify:

$9\sqrt{7} - 6\sqrt{7}$

The surds need to be the same to be able to add or subtract them.

$9 - 6 \rightarrow 3\sqrt{7}$

Link: Similar to $9x - 6x = 3x$

★★

Simplify:

$\sqrt{18} + 5\sqrt{2}$

In some cases, the surds might need to be simplified by operating with them.

$$\sqrt{9 \times 2} + 5\sqrt{2}$$
$$\sqrt{9} \times \sqrt{2} + 5\sqrt{2}$$
$$3\sqrt{2} + 5\sqrt{2}$$

same!

$3 + 5 \rightarrow 8\sqrt{2}$

★★★

Simplify:

$\sqrt{5} \times \sqrt{2}$

Combine to a single surd

$$\sqrt{5 \times 2}$$

$\sqrt{10} \leftarrow 5 \times 2$

★★★★

Simplify:

$\sqrt{21} \div \sqrt{3}$

Similar to multiplying surds, combine to a single surd

$\sqrt{\frac{21}{3}} = \sqrt{7} \leftarrow 21 \div 3$

★★★★★

Simplify:

$$\frac{\sqrt{10} + \sqrt{90}}{\sqrt{5}}$$ Simplify the top first

$$\sqrt{10} + \sqrt{9 \times 10} = \sqrt{10} + \sqrt{9}\sqrt{10} = \sqrt{10} + 3\sqrt{10} = 4\sqrt{10}$$

Substitute back in and simplify:

$$\frac{4\sqrt{10}}{\sqrt{5}} = 4\frac{\sqrt{10}}{\sqrt{5}} = 4\sqrt{\frac{10}{5}} = 4\sqrt{2}$$

★★★★★★

Simplify:

$(3 - \sqrt{2})(6 + \sqrt{2})$

$3 \times 6 \quad 3 \times \sqrt{2} \quad -\sqrt{2} \times 6 \quad -\sqrt{2} \times \sqrt{2}$

$18 + 3\sqrt{2} - 6\sqrt{2} - 2$

Simplify by collecting like terms

$16 - 3\sqrt{2}$

Rationalise the denominator

Fact sheet

★

What could you multiply $\sqrt{13}$ by to make it 13?

$\sqrt{13}$

$\sqrt{13} \times \sqrt{13} = \sqrt{169} = 13$

or

Since √13 is the same as $13^{\frac{1}{2}}$

$13^{1/2} \times 13^{1/2} = 13^1$

★★

Rationalise the denominator:

$$\frac{1}{\sqrt{3}}$$

In other words, remove the surd from the denominator

$$\frac{1}{\sqrt{3}} \overset{\times\sqrt{3}}{=} \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$$

$\times\sqrt{3}$

Equivalent

★★★

Rationalise the denominator:

$$\frac{130}{\sqrt{13}}$$

$$\frac{130}{\sqrt{13}} \overset{\times\sqrt{13}}{=} \frac{130\sqrt{13}}{\sqrt{13} \times \sqrt{13}} = \frac{130\sqrt{13}}{13}$$

$\times\sqrt{13}$

Simplify

$130 \div 13 = 10\sqrt{13}$

★★★★

Rationalise the denominator:

$$\frac{5 + \sqrt{3}}{\sqrt{3}}$$

$\times\sqrt{3}$ Expand

$$\frac{5 + \sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}(5 + \sqrt{3})}{\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{3} + 3}{3}$$

$\times\sqrt{3}$

★★★★★

Difference of 2 squares

Rationalise the denominator:

$$\frac{19}{7 + \sqrt{7}}$$

$\times(7 - \sqrt{7})$

$$\frac{19}{7 + \sqrt{7}} = \frac{19(7 - \sqrt{7})}{(7 + \sqrt{7})(7 - \sqrt{7})} = \frac{133 - 19\sqrt{7}}{49 + 7\sqrt{7} - 7\sqrt{7} - 7}$$
$$= \frac{133 - 19\sqrt{7}}{42}$$

$\times(7 - \sqrt{7})$

★★★★★★

Rationalise the denominator:

$$\frac{392 - \sqrt{392}}{2 - \sqrt{2}}$$

$\sqrt{392} = \sqrt{196}\sqrt{2} = 14\sqrt{2}$

$$\frac{392 - \sqrt{392}}{2 - \sqrt{2}} = \frac{(392 - \sqrt{392})(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})}$$
$$= \frac{784 + 392\sqrt{2} - 28\sqrt{2} - 28}{4 + 2\sqrt{2} - 2\sqrt{2} - 2}$$
$$= (756 + 364\sqrt{2})/2 = 378 + 182\sqrt{2}$$

Sine and cosine rules

Practise questions

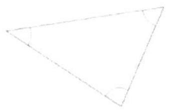
Scan for answers

Scan for copilot

Scan for more questions

★

When using trig with non-right angled triangles, label:



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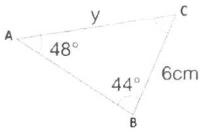
Calculate y to 1dp.



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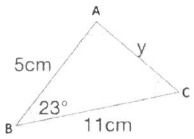
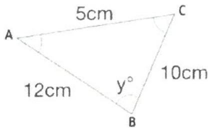
Calculate y to 1dp.



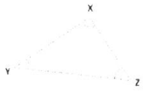
★★★★

Calculate y to 1dp.



★★★★★

If XYZ = 78°, XZY = 76° and YZ = 11cm, is X closer to Y or Z?:



Operate with surds

Practise questions

Scan for answers

Scan for copilot

Scan for more questions

★

Simplify:

$11\sqrt{5} + 3\sqrt{5}$

★★

Simplify:

$\sqrt{44} + 4\sqrt{11}$

★★★

Simplify:

$\sqrt{11} \times \sqrt{7}$

★★★★

Simplify:

$\sqrt{14} \div \sqrt{2}$

★★★★★

Simplify:

$\frac{\sqrt{21} + \sqrt{84}}{\sqrt{3}}$

★★★★★★

Simplify:

$(7 - \sqrt{2})(4 + \sqrt{2})$

Rationalise the denominator

Practise questions

Scan for answers

Scan for copilot

Scan for more questions

★

What could you multiply $\sqrt{5}$ by to make it 5?

★★

Rationalise the denominator:

$\frac{1}{\sqrt{19}}$

★★★

Rationalise the denominator:

$\frac{70}{\sqrt{7}}$

★★★★

Rationalise the denominator:

$\frac{5 + \sqrt{13}}{\sqrt{13}}$

★★★★★

Rationalise the denominator:

$\frac{7}{11 + \sqrt{11}}$

★★★★★★

Rationalise the denominator:

$\frac{162 - \sqrt{162}}{2 - \sqrt{2}}$

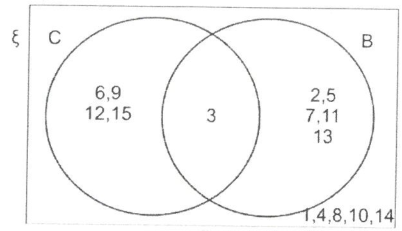
Calculate conditional probabilities

I do, you do example

Scan for answers

I DO

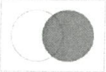
Calculate P(C∩B), P(B) and P(C|B) from:



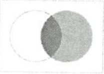
Probability of C and B

$P(C \cap B) = \frac{1}{15}$

Probability of B



$P(B) = \frac{6}{15}$



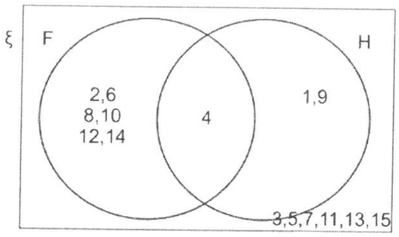
$P(C|B) = \frac{P(C \cap B)}{P(B)}$

$= \frac{1}{15} \div \frac{6}{15} = \frac{1}{6}$

Probability of C given B

YOU DO

Calculate P(F∩H), P(H) and P(F|H) from:



Calculate conditional probabilities

Practise questions

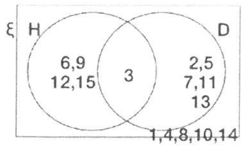
Scan for answers

Scan for copilot

Scan for more questions

★

Calculate P(H∩D), P(D) and P(H|D) from:



★★

Calculate P(A∩G), P(G) and P(A|G) from:

ξ = {1 to 15}

A = {multiples of 3}

G = {square numbers}

★★★

Calculate P(letter I∩vowel), P(vowel) and P(letter I|vowel) in the word FIBONACCI

★★★★

Lou & Kev play hockey. 63% are home games. Lou wins 74% of the home games but only 30% away. Find P(played at home|Lou wins).

Mathematics
Transition to Year 11

Higher Plus

Solutions

Algebraic fractions

I do, you do example

I DO

Simplify:

$$\frac{d^2}{d^2 + 14d + 48} \times \frac{d + 6}{d^3}$$

With algebraic fractions, you follow the normal techniques for operating with numeric fractions, but also use factorising to simplify where possible.

$$\frac{d^2}{(d^2+14d+48)} \times \frac{d+6}{d^3}$$

Factorise

$$= \frac{d^2}{(d+6)(d+8)} \times \frac{d+6}{d^3}$$

Cancel common factors

$$= \frac{\cancel{d^2}\cancel{(d+6)}}{\cancel{(d+6)}(d+8)d^{\overset{1}{\cancel{3}}}} = \frac{1}{d(d+8)}$$

YOU DO

Simplify:

$$\frac{a^2 + 15a + 56}{a^2 + 5a - 14} \times \frac{a - 2}{a + 7}$$

Factorise

$$\frac{(a+7)(a+8)}{(a+7)(a-2)} \times \frac{a-2}{a+7}$$

$$= \frac{\cancel{(a+7)}(a+8)\cancel{(a-2)}}{\cancel{(a+7)}\cancel{(a-2)}(a+7)}$$

$$= \frac{a+8}{a+7}$$

Change the subject

I do, you do example

I DO

Make c the subject:

$cb - a = e$

To make c the subject we use inverse operations to isolate c on one side of the equals sign.

$cb - a = e$

$+a \quad +a$

$cb = e + a$

$\div b \quad \div b$

$c = \frac{e+a}{b}$

When something isn't divisible, leave the answer as a fraction

YOU DO

Make e the subject:

$ef + b = a$

$ef + b = a$

$-b \quad -b$

$ef = a - b$

$\div f \quad \div f$

$e = \frac{a-b}{f}$

Find the equation of a perpendicular line

I do, you do example

I DO

Write down the equation of a line perpendicular to $y=-3x+5$ that passes through the point (3,7). Give your answer in the form ax+by=c.

A perpendicular line has a gradient that is the negative reciprocal of the original gradient. When multiplied together, the product of the gradients is -1.

$m = -3$ Gradient of original line

$-\frac{1}{m} = \frac{1}{3}$ Gradient of perpendicular line

We know the line passes through (3,7) so if $y = \frac{1}{3}x + c$, then:

$7 = \frac{1}{3}(3) + c$

$7 = 1 + c$ therefore $c = 6$

$y = \frac{1}{3}x + 6 \Rightarrow 3y = x + 18 \Rightarrow \overset{ax \; + \; by \; = \; c}{-x + 3y = 18}$

YOU DO

Write down the equation of a line perpendicular to $y=8-6x$ that passes through the point (6,-8). Give your answer in the form ax+by=c.

$m = -6$ so $-\frac{1}{m} = \frac{1}{6}$

$y = \frac{1}{6}x + c$

$-8 = \frac{1}{6}(6) + c$

$-8 = 1 + c$ so $c = -9$

$y = \frac{1}{6}x - 9$

$6y = x - 54$

$-x + 6y = -54$

or $x - 6y = 54$

Algebraic fractions

★

Simplify:

$$\frac{48a^7g^4}{6a^5g^7}$$

$$\equiv \frac{48}{6} \times \frac{a^7}{a^5} \times \frac{g^4}{g^7}$$ subtract the powers

$$\equiv 8a^2g^{-3}$$

★★

Calculate:

$$\frac{a}{d} \div \frac{e}{h}$$ Reciprocal

$$\equiv \frac{a}{d} \times \frac{h}{e}$$

$$\equiv \frac{ah}{de}$$

★★★

Simplify:

$$\frac{g^2 - 2g - 15}{gf - 5f}$$ Factorise

$$\equiv \frac{(g+3)\cancel{(g-5)}}{f\cancel{(g-5)}}$$

$$\equiv \frac{g+3}{f}$$

★★★★

Simplify:

$$\frac{7c}{c-3} \overset{x}{\times} \frac{c}{7}$$ x

$$\equiv \frac{\cancel{7}c^2}{\cancel{7}(c-3)}$$

$$\equiv \frac{c^2}{c-3}$$

★★★★★

Simplify:

Seek the common denominator

$$\frac{3}{c+7} - \frac{6}{2c+3}$$

$$\equiv \frac{3(2c+3)}{(c+7)(2c+3)} - \frac{6(c+7)}{(c+7)(2c+3)}$$

$$\equiv \frac{6c+9-6c-42}{(c+7)(2c+3)} \equiv \frac{-33}{(c+7)(2c+3)}$$

★★★★★★

$$\frac{12(2f+13)-9(f+17)}{(f+17)(2f+13)} = 3$$

$$\frac{24f+156-9f-153}{2f^2+13f+34f+221} = 3$$

$$\frac{15f+3}{2f^2+47f+221} = 3$$

$$15f+3 = 6f^2+141f+663$$

$$6f^2+126f+660 = 0$$

$$f^2+21f+110=0 \rightarrow (f+10)(f+11)=0$$

$$f=-10 \quad f=-11$$

Change the subject

★

Make c the subject:

$$c + e = b$$

$$-e \quad -e$$

$$c = b - e$$

★★

Make d the subject:

$$dc - f = b$$

$$+f \quad +f$$

$$dc = b + f$$

$$\div c \quad \div c$$

$$d = \frac{b+f}{c}$$

★★★

Make c the subject:

$$\times e \quad \frac{cb-a}{e} = d \quad \times e$$

$$cb - a = de$$

$$+a \quad +a$$

$$cb = de + a$$

$$\div b \quad \div b$$

$$c = \frac{de+a}{b}$$

★★★★

Make a the subject:

$$\times b \quad \frac{\sqrt{a} - c}{b} = e \quad \times b$$

$$\sqrt{a} - c = eb$$

$$+c \quad +c$$

$$\sqrt{a} = eb + c$$

$$a = (eb+c)^2$$

★★★★★

Make r the subject:

$$A = \pi r^2$$

$$\div\pi \quad \div\pi$$

$$\frac{A}{\pi} = r^2$$

$$\sqrt{\frac{A}{\pi}} = r$$

★★★★★★

Make c the subject:

$$e = ac + 2c$$ factorise

$$e = c(a+2)$$

$$\frac{e}{a+2} = c$$

Find the equation of a perpendicular line

★

The gradient of a line is

$$\frac{4}{7}$$

What is the gradient of a perpendicular line?

$$-\frac{7}{4}$$

★★

Line J passes through (-9,-14) and (-14,-11). Line Q passes through (2,-8) and (3,11). Is line J perpendicular to line Q?

Line j gradient $= \frac{-11-(-14)}{-14-(-9)} = -\frac{3}{5}$

Line Q gradient $= \frac{11-(-8)}{3-2} = \frac{19}{1}$

$-\frac{3}{5} \times \frac{19}{1} = -\frac{57}{5}$ ← Not -1 so no

★★★

Write down the equation of a line perpendicular to $y = 5x - 6$.

$m = 5$ so $-\frac{1}{m} = -\frac{1}{5}$

Any equation that fits $y = -\frac{1}{5}x + c$ would work here so

$$y = -\frac{1}{5}x + 2$$

★★★★

Prove that the following two lines are perpendicular:

$$y = \boxed{2}x + 5$$

$$y = \boxed{-1/2}x + 8$$

$$2 \times -\frac{1}{2} = -1$$

Product of the gradients is -1

★★★★★

The equations of 2 lines are given below:

$$y = \boxed{7}x + 3$$

$$y = 8 + \boxed{1/7}x$$

Are they perpendicular?

$$7 \times \frac{1}{7} = 1$$

No they are not perpendicular since the product is not -1.

★★★★★★

If $m=-5$ $\quad -\frac{1}{m} = \frac{1}{5}$

$$y = \frac{1}{5}x + c$$

$$-2 = \frac{1}{5}(-5) + c$$

$$-2 = -1 + c \quad \text{so } c=-1$$

$$y = \frac{1}{5}x - 1$$

$$5y = x - 5$$

$$-x+5y = -5 \quad \text{or} \quad x-5y = 5$$

Trigonometry graphs

Fact sheet

Complete the square

I do, you do example

I DO

Write the following expression in completed square form:

$x^2 + 4x - 1$

The completed square form is:

$(x + d)^2 + e$

$\frac{1}{2}b$ ↗ ↖ $c - d^2$

So for a quadratic above that is written in the form $ax^2 + bx + c$:

a = 1, b = 4 and c = -1

$d = \frac{1}{2}(4) = 2$

$e = -1 - (2)^2 = -1 - 4 = -5$

$(x + 2)^2 - 5$

From this form, we can also get the turning point of the quadratic: (-2, -5)

YOU DO

Write the following expression in completed square form:

$x^2 - 6x - 2$

$a = 1 \quad b = -6 \quad c = -2$

$d = \frac{1}{2}(-6) = -3$

$e = -2 - (-3)^2$

$= -2 - 9 = -11$

$(x - 3)^2 - 11$

Solve a quadratic by completing the square

I do, you do example

I DO

Solve the following quadratic by completing the square (leave in exact form):

$x^2 + 2x - 2 = 0$

Complete the square

$(x + 1)^2 - 2 - (1)^2 = 0$

$(x + 1)^2 - 2 - 1 = 0$

$(x + 1)^2 - 3 = 0$

Begin to rearrange to make x the subject

$(x + 1)^2 = 3$

$x + 1 = \pm\sqrt{3}$

$x = -1 \pm \sqrt{3}$

There are 2 answers in this case due to the + and - of the root of 3

YOU DO

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 4x - 4 = 0$

$(x - 2)^2 - 4 - (-2)^2 = 0$

$(x - 2)^2 - 4 - 4 = 0$

$(x - 2)^2 - 8 = 0$

$(x - 2)^2 = 8$

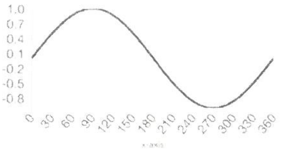
$x - 2 = \pm\sqrt{8}$

$x = 2 \pm \sqrt{8}$

Trigonometry graphs

★

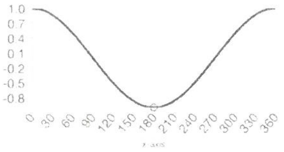
What is the equation of the following graph?



$y = \sin x°$

★★

State the coordinates of the point on the graph:

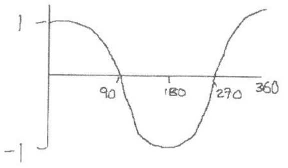


-1

★★★

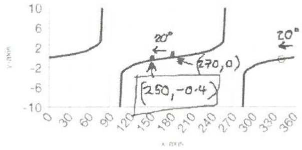
Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360$.

x	0	60	90	120	180
y	1	½	0	-½	-1



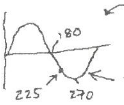
★★★★

When x = 340°, tan$x$ ≈ -0.4. Find another value of x that satisfies tan$x$ ≈ -0.4.



★★★★★

When x = 225°, sin$x$ ≈ -0.7. Find another value of x that satisfies sin$x$ ≈ -0.7.



Draw a quick sketch

315° due to the symmetry between 180° and 360°

Complete the square

★

Find the value of a in the following:

$x^2 + 8x + 19 = (x + 4)^2 + a$

$a=1 \quad b=8 \quad c=19$

$e = 19 - 4^2$

$= 19 - 16 = 3$

$a = 3$

★★

Find the value of a in the following:

$x^2 - 18x + 88 = (x - 9)^2 + a$

$a=1 \quad b=-18 \quad c=88$

$e = 88 - (-9)^2$

$= 88 - 81 = 7$

$a = 7$

★★★

Write the following expression in completed square form:

$x^2 - 2x - 7$

$a=1 \quad b=-2 \quad c=-7$

$d = ½(-2) = -1$

$e = -7 - (-1)^2$

$= -7 - 1 = -8$

$(x - 1)^2 - 8$

★★★★

Write the following expression in completed square form:

$x^2 + 7x + 7$

$a=1 \quad b=7 \quad c=7$

$d = ½(7) = 3.5$

$e = 7 - (3.5)^2$

$= 7 - 12.25 = -5.25$

$(x + 3.5)^2 - 5.25$

★★★★★

Find the minimum point of the following function:

$f(x) = x^2 - 6x - 3$

$a=1 \quad b=-6 \quad c=-3$

$d = ½(-6) = -3$

$e = -3 - (-3)^2 = -3 - 9$

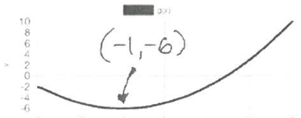
$= -12$

$(x - 3)^2 - 12$

Turning point at $(3, -12)$

★★★★★★

$f(x) = x^2$ has translated to $g(x)$. Identify $g(x)$.



So $(x + 1)^2 - 6$

$g(x) = x^2 + 2x - 5$

Solve a quadratic by completing the square

★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 + 6x - 8 = 0$

$(x + 3)^2 + a = 0 \quad -8 - (3)^2$

$(x + 3)^2 - 17 = 0$

$(x + 3)^2 = 17$

$x + 3 = \pm\sqrt{17}$

$x = -3 \pm \sqrt{17}$

★★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 4x - 1 = 0$

$(x - 2)^2 + a = 0 \quad -1 - (-2)^2$

$(x - 2)^2 - 5 = 0$

$(x - 2)^2 = 5$

$x - 2 = \pm\sqrt{5}$

$x = 2 \pm \sqrt{5}$

★★★

Solve the following quadratic by completing the square (leave in exact form):

$x^2 - 2x - 4 = 0$

$(x - 1)^2 - 5 = 0$

$(x - 1)^2 = 5$

$x - 1 = \pm\sqrt{5}$

$x = 1 \pm \sqrt{5}$

★★★★

$x^2 - 3x - 5 = 0$

$(x - 1.5)^2 - 7.25 = 0$

$(x - 1.5)^2 = 7.25$

$x - 1.5 = \pm\sqrt{7.25}$

$x = 1.5 \pm \sqrt{7.25}$

★★★★★

$5(x^2 - x) - 3 = 0$

$5[(x - 0.5)^2 - 0.25) - 3 = 0$

$5(x - 0.5)^2 - 1.25 - 3 = 0$

$5(x - 0.5)^2 - 4.25 = 0$

$5(x - 0.5)^2 = 4.25$

$(x - 0.5)^2 = 0.85$

$x - 0.5 = \pm\sqrt{0.85}$

$x = 0.5 \pm \sqrt{0.85}$

$x = 1.42 \qquad x = -0.42$

★★★★★★

$8(x^2 + \frac{17}{8}x) - 18 = 0$

$8((x + \frac{17}{16})^2 - (\frac{17}{16})^2) - 18 = 0$

$8((x + \frac{17}{16})^2 - \frac{289}{256}) - 18 = 0$

$8(x + \frac{17}{16})^2 - \frac{289}{32} - 18 = 0$

$8(x + \frac{17}{16})^2 - 27\frac{1}{32} = 0$

$8(x + \frac{17}{16})^2 = 27\frac{1}{32}$

$(x + \frac{17}{16})^2 = 216\frac{1}{4}$

$x + \frac{17}{16} = \pm\sqrt{216\frac{1}{4}}$

$x = -\frac{17}{16} \pm \sqrt{216\frac{1}{4}}$

$x = 0.78$

$x = -2.90$

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Solve a quadratic by using the formula

I do, you do example

I DO

Using the quadratic formula, solve the following equation (to 2 d.p.):

$5x^2 - 13x + 3 = 0.$

a b c

A quadratic can be solved by using the quadratic formula when it is written in the form $ax^2 + bx + c = 0$ and the discriminant $b^2 - 4ac \geq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4 \times 5 \times 3}}{2 \times 5}$$

Simplify

$$x = \frac{13 \pm \sqrt{169 - 60}}{10} = \frac{13 \pm \sqrt{109}}{10}$$

Evaluate

$$x = \frac{13 + \sqrt{109}}{10} \approx 2.34 \qquad x = \frac{13 - \sqrt{109}}{10} \approx 0.26$$

YOU DO

Using the quadratic formula, solve the following equation (to 2 d.p.):

$10x^2 - 3x - 9 = 0.$

a b c

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 10 \times (-9)}}{2 \times 10}$$

$$x = \frac{3 \pm \sqrt{9 + 360}}{20} = \frac{3 \pm \sqrt{369}}{20}$$

$$x = \frac{3 + \sqrt{369}}{20} \qquad x = \frac{3 - \sqrt{369}}{20}$$

$$\approx 1.11 \qquad \approx -0.81$$

Solve quadratic inequalities

I do, you do example

I DO

Solve:

$x^2 - 11x + 28 \leq 0$

Solve the quadratic inequality like a quadratic equation

Factorising gives:

$y \leq 0$

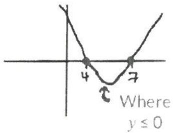
$(x - 4)(x - 7) \leq 0$

So when $x = 4$, $y = 0$

And when $x = 7$, $y = 0$

This gives the points of intersection with the x-axis

Sketch the inequality graph:



Where $y \leq 0$

So:

$4 \leq x \leq 7$

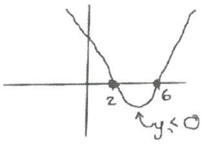
YOU DO

Solve:

$x^2 - 8x + 12 \leq 0$

$(x - 2)(x - 6) \leq 0$

$x = 2$ when $y = 0$ and $x = 6$ when $y = 0$



So $2 \leq x \leq 6$

Graphical inequalities

Fact sheet

Solve a quadratic by using the formula

★

Substitute a = 2, b = 10 and c = 2 into the formula:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4\times2\times2}}{2\times2}$$

★★

Substitute a = 9, b = -6 and c = -9 into the formula:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4\times9\times(-9)}}{2\times9}$$

$$= \frac{6 \pm \sqrt{36 + 324}}{18}$$

★★★

Tia correctly used the quadratic formula to get:

$$x = \frac{\overset{-b}{10} \pm \sqrt{(100 + \overset{-4ac}{468})}}{\underset{2a}{18}}$$

Write down the quadratic equation Tia was solving.

$-b = 10$ $2a = 18$

$b = -10$ $a = 9$

$-4ac = 468$

$-4\times9\times c = 468$

$-36c = 468$

$c = -13$

$9x^2 - 10x - 13 = 0$

★★★★

Use the $\underset{b^2-4ac}{\text{discriminant}}$ to determine how many solutions the following quadratic will have:

$-13x^2 + 9x - 13 = 0.$

$9^2 - 4\times-13\times-13$

$81 - 676 = -595$ ← Since this is negative there are no solutions

★★★★★

Using the quadratic formula, solve the following equation (to 2 d.p.):

$-7x^2 + 1x + 11 = 0.$

$$x = \frac{-1 \pm \sqrt{1^2 - 4\times(-7)\times11}}{2\times-7}$$

$$= \frac{-1 \pm \sqrt{1 + 308}}{-14}$$

$x \approx -1.18$

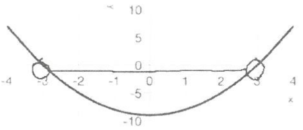
$x \approx 1.33$

Solve quadratic inequalities

★

f(x) = x^2 - 9 is shown.

Solve f(x) < 0.

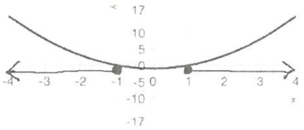


$-3 < x < 3$

★★

f(x) = x^2 - 1 is shown.

Solve f(x) ≥ 0.



$x \leq -1$

$x \geq 1$

★★★

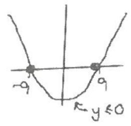
Solve:

$x^2 - 81 \leq 0$

$(x+9)(x-9) \leq 0$

$x = -9$ $y = 0$

$x = 9$ $y = 0$



So $-9 \leq x \leq 9$

★★★★

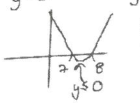
Solve:

$x^2 - 15x + 56 \leq 0$

$(x-7)(x-8) \leq 0$

$x = 7$ $y = 0$

$x = 8$ $y = 0$



So $7 \leq x \leq 8$

★★★★★

Solve:

$3x^2 + 5x + 2 \leq 0$

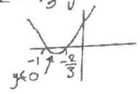
$3x^2 + 3x + 2x + 2 \leq 0$

$3x(x+1) + 2(x+1) \leq 0$

$(3x+2)(x+1) \leq 0$

$x = -\frac{2}{3}$ $y = 0$

$x = -1$ $y = 0$



So $-1 \leq x \leq -\frac{2}{3}$

★★★★★★

Solve:

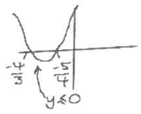
$12x^2 \leq -31x - 20$

$12x^2 + 31x + 20 \leq 0$

$(3x+4)(4x+5) \leq 0$

$x = -\frac{4}{3}$ $y = 0$

$x = -\frac{5}{4}$ $y = 0$

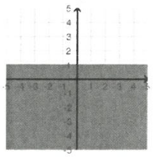


So $-\frac{4}{3} \leq x \leq -\frac{5}{4}$

Graphical inequalities

★

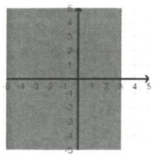
Identify the inequality that satisfies the given region:



$y \leq 1$

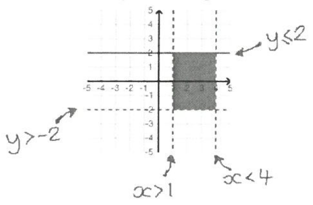
★★

Answer:



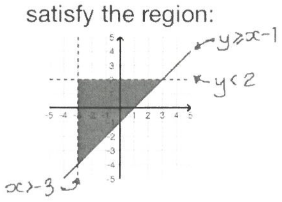
★★★

Identify the inequalities that satisfy the region:



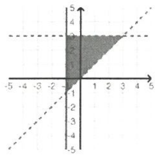
★★★★

Identify the inequalities that satisfy the region:



★★★★★

Answer:



(-1, 0) (-1, 1) (-1, 2) (0, 1) (0, 2) (1, 2)

Understand quadratic sequences

I do, you do example

I DO

Find the nth term for the following sequence:

8, 13, 20, 29, ...

5 7 9

2 2

Unlike linear sequences, quadratic sequences have a second line of common differences. The rule always comes in the form $ax^2 + bx + c$, where a is half the second line of differences.

$a = \frac{1}{2}(2) = 1$

The rule is n^2+bn+c. ← Notice if you take n away, you get bn + c (a linear rule)

n^2+bn+c	8	13	20	29
n^2	1	4	9	16
$bn+c$	7	9	11	13 ← 2n + 5

So the rule is n^2+2n+5

YOU DO

Find the nth term for the following sequence:

5, 10, 17, 26, ...

5 7 9

2 2

$a = \frac{1}{2}(2) = 1$

The rule is n^2+bn+c

n^2+bn+c	5	10	17	26
n^2	1	4	9	16
$bn+c$	4	6	8	10 → 2n+2

So the rule is n^2+2n+2

Solve non linear simultaneous equations

I do, you do example

I DO

Solve for x and y:

① $x^2 + y^2 = 34$

② $x + y = 2$

Label the equations to help structure your work

First we need to rearrange (2) for either x or y:

③ $y = 2 - x$

Substitute equation (3) into equation (1):

Expand and simplify

$x^2 + (2-x)^2 = 34$

$x^2 + 4 - 4x + x^2 = 34$

$2x^2 - 4x - 30 = 0$ ← Make it equal to 0

Solve the quadratic using a suitable method:

Factorise

$2x^2 + 6x - 10x - 30 = 0$

$2x(x+3) - 10(x+3) = 0$

$(2x-10)(x+3) = 0$

$x = 5 \quad x = -3$

Substitute both x into equation (3) to find y:

$y = 2-5 = -3 \quad y = 2--3 = 5$

YOU DO

Solve for x and y:

① $x^2 + y^2 = 37$

② $x + y = 7$

$y = 7 - x$

$x^2 + (7-x)^2 = 37$

$x^2 + 49 - 14x + x^2 = 37$

$2x^2 - 14x + 49 = 37$

$2x^2 - 14x + 12 = 0$

$2x^2 - 2x - 12x + 12 = 0$

$2x(x-1) - 12(x-1) = 0$

$(2x-12)(x-1) = 0$

$x = 6 \quad x = 1$

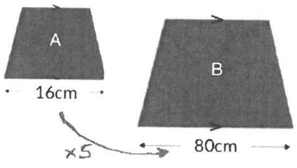
$y = 7-6 = 1 \quad y = 7-1 = 6$

Area/Volume scale factors

I do, you do example

I DO

A and B are similar shapes. The area of A is 80cm². Find the area of B?



Similar shapes are 2 of the same shape but different in size.

Their lengths are linked by a scale factor.
Their areas are linked by (scale factor)².
Their volumes are linked by (scale factor)³.

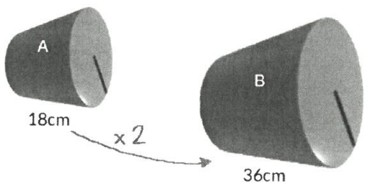
Scale factor $= \frac{80}{16} = 5$

Area scale factor $= 5^2 = 25$

So area of shape B $= 80 \times 25 = 2000\text{cm}^2$

YOU DO

A and B are similar. The volume of B is 720cm³. Find the volume of A?



Scale factor $= \frac{36}{18} = 2$

Volume scale factor $= 2^3 = 8$

So volume of shape A $= 720 \div 8 = 90\text{cm}^3$

Understand quadratic sequences

★

Continue the sequence for the next 2 terms:

5, 15, 31, 53, 81, 115

10 16 22 28 34

6 6 6 6

★★

Find the nth term rule for the following sequence:

6, 24, 54, 96, ...

18 30 42

12 12

$a = \frac{1}{2}(12) = 6$

$6n^2+bn+c$ 6 24 54 96

$6n^2$ 6 24 54 96

this is the rule

★★★

Find the nth term for the following sequence:

10, 18, 28, 40, ...

8 10 12

2 2

$a = \frac{1}{2}(2) = 1$

n^2+bn+c 10 18 28 40

n^2 1 4 9 16

$bn+c$ 9 14 19 24

$5n+4$

The rule is:

n^2+5n+4

★★★★

Find the nth term for the following sequence:

5, 12, 25, 44, ...

7 13 19

6 6

$a = \frac{1}{2}(6) = 3$

$3n^2+bn+c$ 5 12 25 44

$3n^2$ 3 12 27 48

$bn+c$ 2 0 -2 -4

$-2n+4$

The rule is $3n^2-2n+4$

★★★★★

Find the nth term for the following sequence:

-12, -30, -58, -96, ...

-18 -28 -38

-10 -10

$a = \frac{1}{2}(-10) = -5$

$-5n^2+bn+c$ -12 -30 -58 -96

$-5n^2$ -5 -20 -45 -80

$bn+c$ -7 -10 -13 -16

$-3n-4$

The rule is $-5n^2-3n-4$

★★★★★★

Find the 80th term in the following sequence:

-7, -19, -37, -61, ...

-12 -18 -22

-6 -6

$a = \frac{1}{2}(-6) = -3$

$-3n^2+bn+c$ -7 -19 -37 -61

$-3n^2$ -3 -12 -27 -48

$bn+c$ -4 -7 -10 -13

$-3n-1$

The rule is $-3n^2-3n-1$

The 80th term is $-3(80)^2-3\times80-1$

$= -19441$

Solve non linear simultaneous equations

★

Solve for x and y:

$y = x^2$

$y = -7x - 12$

$x^2 = -7x-12$

$x^2+7x+12=0$

$(x+3)(x+4)=0$

$x=-3$ $x=-4$

$y=(-3)^2$ $y=(-4)^2$

$=9$ $=16$

★★

Solve for x and y:

$y = x^2 + 3x - 24$

$y = 3x + 25$

$x^2+3x-24 = 3x+25$

$x^2-49=0$

$(x-7)(x+7)=0$

$x=7$ $x=-7$

$y=3\times7+25$ $y=3\times-7+25$

$=46$ $=4$

★★★

Solve for x and y:

$xy = -12$

$y = x + 7$

$x(x+7) = -12$

$x^2+7x=-12$

$x^2+7x+12=0$

$(x+3)(x+4)=0$

$x=-3$ $x=-4$

$y=-3+7$ $y=-4+7$

$=4$ $=3$

★★★★

Answer:

$x = 4$, $y = -1$ and

$x = -1$, $y = 4$

$y=3-x$

$x^2+(3-x)^2=17$

$x^2+9-6x+x^2=17$

$2x^2-6x+9=17$

$2x^2-6x-8=0$

$2x^2-8x+2x-8=0$

$2x(x-4)+2(x-4)=0$

$(2x+2)(x-4)=0$

$x=-1$ $x=4$

$y=3--1$ $y=3-4$

$=4$ $=-1$

★★★★★

Solve for x and y (to 2dp):

$x^2 + y^2 = 27$

$x + y = 6$

$y=6-x$

$x^2+(6-x)^2=27$

$x^2+36-12x+x^2=27$

$2x^2-12x+36=27$

$2x^2-12x+9=0$

Use the quadratic formula to solve

$x = \frac{12 \pm \sqrt{(-12)^2-4\times2\times9}}{2\times2}$

$= \frac{12\pm\sqrt{144-72}}{4}$

$= \frac{12\pm\sqrt{72}}{4} = \frac{12\pm6\sqrt{2}}{4}$

$x=5.1213203$ $x=0.87868$

$y=6-5.1213203$ $y=6-0.87868$

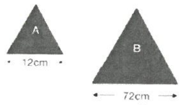
$y=0.87868$ $y=5.1213203$

$x=5.12, y=0.88$ $x=0.88$ $y=5.12$

Area/Volume scale factors

★

A and B are similar shapes. The area of A is 36cm^2. Find the area of B?



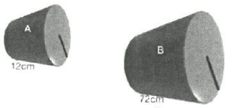
Scale factor $= \frac{72}{12} = 6$

Area scale factor $= 6^2 = 36$

Area of B $= 36\times36 = 1296\text{cm}^2$

★★

A and B are similar. The volume of A is 60cm^3. Find the volume of B?



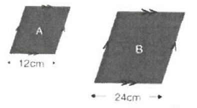
Scale factor $= \frac{72}{12} = 6$

Volume scale factor $= 6^3 = 216$

Volume of B $= 60\times216 = 12960\text{cm}^3$

★★★

A and B are similar shapes. The area of B is 192cm^2. Find the area of A?



Scale factor $= \frac{24}{12} = 2$

Area scale factor $= 2^2 = 4$

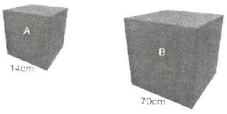
Area of A $= 192 \div 4 = 48\text{cm}^2$

★★★★

Scale factor $= \frac{70}{14} = 5$

Volume scale factor $= 5^3 = 125$

Volume of A $= \frac{8750}{125} = 70\text{cm}^3$

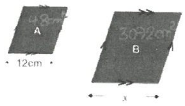


★★★★★

Area scale factor $= \frac{3072}{48} = 64$

Scale factor $= \sqrt{64} = 8$

$x = 12\times8 = 96\text{cm}$

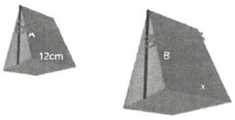


★★★★★★

Volume scale factor $= \frac{3840}{60} = 64$

Scale factor $= \sqrt[3]{64} = 4$

$x = 12\times4 = 48\text{cm}$



Sine and cosine rules

Fact sheet

Operate with surds

Fact sheet

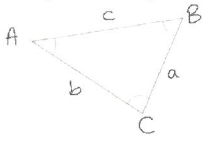
Rationalise the denominator

Fact sheet

Sine and cosine rules

★

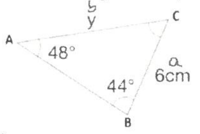
When using trig with non-right angled triangles, label:



This is one possible option. The key thing to check is that opposite side and angle are the same letter.

★★

Calculate y to 1dp.

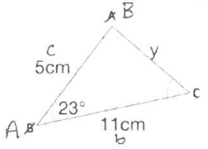


$\frac{6}{\sin 48} = \frac{y}{\sin 44}$

$y = \frac{6}{\sin 48} \times \sin 44 \approx 5.6cm$

★★★

Calculate y to 1dp.



$a^2 = 11^2 + 5^2 - 2 \times 11 \times 5 \times \cos 23$

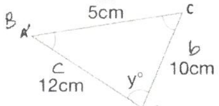
$= 121 + 25 - 110 \cos 23$

$= 146 - 110 \cos 23$

$a = \sqrt{146 - 110 \cos 23} \approx 6.7cm$

★★★★

Calculate y to 1dp.

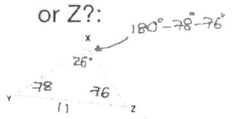


$\cos A = \frac{10^2 + 12^2 - 5^2}{2 \times 10 \times 12} = \frac{100+144-25}{240}$

$A = \cos^{-1}\left(\frac{219}{240}\right) \approx 24.1°$

★★★★★

If XYZ = 78°, XZY = 76° and YZ = 11cm, is X closer to Y or Z?:



XY < XZ so X is closer to Y.

Calculating XY

$\frac{a}{\sin 76} = \frac{11}{\sin 26}$

$a = \frac{11}{\sin 26} \times \sin 76 \approx 24.3$

Calculating XZ

$\frac{a}{\sin 78} = \frac{11}{\sin 26}$

$a = \frac{11}{\sin 26} \times \sin 78 \approx 24.5$

Operate with surds

★

Simplify:

11√5 + 3√5

$14\sqrt{5}$

11+3

★★

Simplify:

√44 + 4√11

simplify

$\sqrt{4 \times 11} + 4\sqrt{11}$

$\sqrt{4} \times \sqrt{11} + 4\sqrt{11}$

$2\sqrt{11} + 4\sqrt{11}$

$6\sqrt{11}$

★★★

Simplify:

√11 x √7

$\sqrt{77}$

11 x 7

★★★★

Simplify:

√14 ÷ √2

$\sqrt{7}$ ← 14÷2

★★★★★

Simplify:

$\frac{\sqrt{21} + \sqrt{84}}{\sqrt{3}}$

Top first

$\sqrt{21} + \sqrt{4 \times 21} = \sqrt{21} + 2\sqrt{21} = 3\sqrt{21}$

$\frac{3\sqrt{21}}{\sqrt{3}} = 3\sqrt{\frac{21}{3}} = 3\sqrt{7}$

★★★★★★

Simplify:

(7 - √2)(4 + √2)

Expand

$28 + 7\sqrt{2} - 4\sqrt{2} - 2$

Simplify

$26 + 3\sqrt{2}$

Rationalise the denominator

★

What could you multiply √5 by to make it 5?

$\sqrt{5}$

★★

Rationalise the denominator:

$\frac{1}{\sqrt{19}}$

$= \frac{\sqrt{19}}{\sqrt{19} \times \sqrt{19}}$

$= \frac{\sqrt{19}}{19}$

★★★

Rationalise the denominator:

$\frac{70}{\sqrt{7}}$

$= \frac{70\sqrt{7}}{\sqrt{7} \times \sqrt{7}}$

$= \frac{70\sqrt{7}}{7}$

$= 10\sqrt{7}$

★★★★

Rationalise the denominator:

$\frac{5 + \sqrt{13}}{\sqrt{13}}$

$\frac{5+\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}(5+\sqrt{13})}{\sqrt{13} \times \sqrt{13}} = \frac{5\sqrt{13}+13}{13}$

★★★★★

Rationalise the denominator:

$\frac{7}{11 + \sqrt{11}}$

$\frac{7}{11+\sqrt{11}} = \frac{7(11-\sqrt{11})}{(11+\sqrt{11})(11-\sqrt{11})} = \frac{77 - 7\sqrt{11}}{121 - 11\sqrt{11} + 11\sqrt{11} - 11}$

$= \frac{77-7\sqrt{11}}{110}$

★★★★★★

Rationalise the denominator:

$\frac{162 - \sqrt{162}}{2 - \sqrt{2}}$

$\sqrt{162}$
$= \sqrt{81}\sqrt{2}$
$= 9\sqrt{2}$

$\frac{162-\sqrt{162}}{2-\sqrt{2}} = \frac{(162-\sqrt{162})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$

$= \frac{324 + 162\sqrt{2} - 18\sqrt{2} - 18}{4 + 2\sqrt{2} - 2\sqrt{2} - 2}$

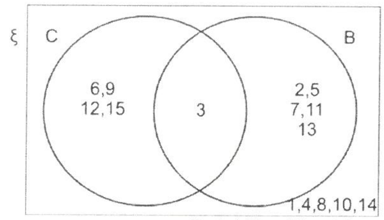
$= \frac{306 + 144\sqrt{2}}{2} = 153 + 72\sqrt{2}$

Calculate conditional probabilities

I do, you do example

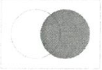
I DO

Calculate P(C∩B), P(B) and P(C|B) from:



Probability of C and B

$P(C \cap B) = \frac{1}{15}$



Probability of B

$P(B) = \frac{6}{15}$

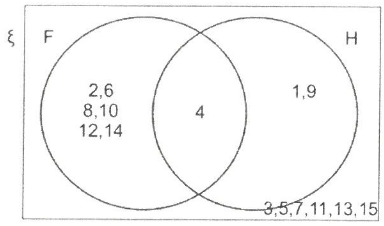
$P(C|B) = \frac{P(C \cap B)}{P(B)}$

$= \frac{1}{15} \div \frac{6}{15} = \frac{1}{6}$

Probability of C given B

YOU DO

Calculate P(F∩H), P(H) and P(F|H) from:



$P(F \cap H) = \frac{1}{15}$

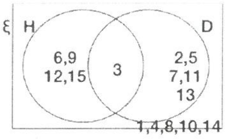
$P(H) = \frac{3}{15} = \frac{1}{5}$

$P(F|H) = \frac{1}{15} \div \frac{1}{5} = \frac{1}{3}$

Calculate conditional probabilities

★

Calculate P(H∩ D), P(D) and P(H|D) from:



$P(H\cap D)=\frac{1}{15}$ $P(H|D)=\frac{1}{15}\div\frac{6}{15}$

$P(D)=\frac{6}{15}$ $=\frac{1}{6}$

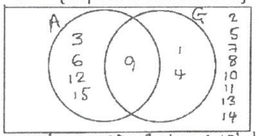
★★

Calculate P(A∩ G), P(G) and P(A|G) from:

ξ = {1 to 15}

A = {multiples of 3}

G = {square numbers}

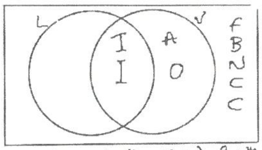


$P(A\cap G)=\frac{1}{15}$ $P(G)=\frac{3}{15}=\frac{1}{5}$ $P(A|G)=\frac{1}{15}\div\frac{1}{5}$

$=\frac{1}{3}$

★★★

Calculate P(letter I∩ vowel), P(vowel) and P(letter I|vowel) in the word FIBONACCI

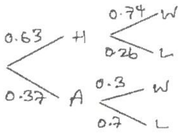


$P(I\cap V)=\frac{2}{9}$ $P(V)=\frac{4}{9}$ $P(I|V)=\frac{2}{9}\div\frac{4}{9}$

$=\frac{2}{4}=\frac{1}{2}$

★★★★

Lou & Kev play hockey. 63% are home games. Lou wins 74% of the home games but only 30% away. Find P(played at home|Lou wins).



$P(H\cap W)=0.63\times0.74=0.4662$

$P(W)=0.63\times0.74+0.37\times0.3$
$=0.5772$

$P(H|W)=0.4662\div0.5772=0.8077$