

Mathematics Transition to Year 11

Higher



Solutions

I do, you do example

Simplify:

$$\frac{d^2}{d^2 + 14d + 48} \times \frac{d+6}{d^3}$$

With algebraic fractions, you follow the normal techniques for operating with numeric fractions, but also use factorising to simplify where possible.

Factorise

$$\frac{d^2}{(d^2 + 14d + 48)} \times \frac{d+6}{d^3}$$

Cancel common factors

$$\frac{d^2}{(d+6)(d+8)} \times \frac{d+6}{d^3}$$

$$\frac{d^2(d+6)}{(d+6)(d+8)d^3} = \frac{1}{d(d+8)}$$

Simplify:

Factorise

$$\frac{a^2 + 15a + 56}{a^2 + 5a - 14} \times \frac{a-2}{a+7}$$

$$\frac{(a+7)(a+8)}{(a+7)(a-2)} \times \frac{a-2}{a+7}$$

$$= \frac{(a+7)(a+8)(a-2)}{(a+7)(a-2)(a+7)}$$

$$= \frac{a+8}{a+7}$$

Change the subject

I do, you do example

Make c the subject:

$$cb - a = e$$

To make c the subject we use inverse operations to isolate c on one side of the equals sign.

$$cb - a = e$$

$$+a \quad +a$$

$$cb = e + a$$

$$\div b \quad \div b$$

$$c = \frac{e+a}{b}$$

When something isn't divisible, leave the answer as a fraction

Make e the subject:

$$ef + b = a$$

$$ef + b = a$$

$$-b \quad -b$$

$$ef = a - b$$

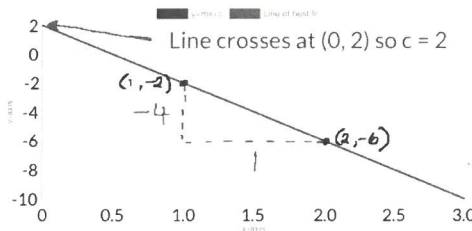
$$\div f \quad \div f$$

$$e = \frac{a-b}{f}$$

Find $y = mx + c$ from a graph

I do, you do example

State $y = mx + c$ for:



Co-ordinates that satisfy the rule $y = mx + c$

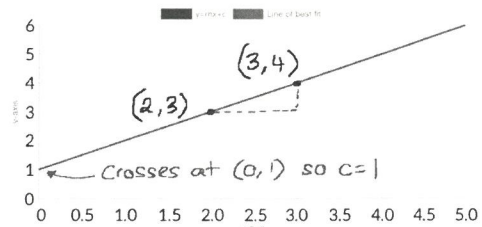
m -intercept (point of intersection of the line with the y -axis)

Gradient (steepness) = $\frac{\text{Change in } y}{\text{Change in } x}$

$$\frac{\text{change in } y}{\text{change in } x} = \frac{-6 - (-2)}{2 - 1} = \frac{-4}{1} = -4$$

$$y = -4x + 2$$

State $y = mx + c$ for:



$$\text{Gradient} = \frac{4 - 3}{3 - 2} = \frac{1}{1} = 1$$

$$y = 1x + 1$$

★
Simplify:
 $\frac{48a^7g^4}{6a^5g^7}$
 $\equiv \frac{48}{6} \times \frac{a^7}{a^5} \times \frac{g^4}{g^7}$ subtract the powers
 $\equiv 8a^2g^{-3}$

★★
Calculate:
 $\frac{a}{d} \div \frac{e}{h}$ Reciprocal
 $\equiv \frac{a}{d} \times \frac{h}{e}$
 $\equiv \frac{ah}{de}$

★★★
Simplify:
 $\frac{g^2 - 2g - 15}{gf - 5f}$ Factorise
 $\equiv \frac{(g+3)(g-5)}{f(g-5)}$
 $\equiv \frac{g+3}{f}$

★★★★
Simplify:
 $\frac{7c}{c-3} \times \frac{c}{7}$
 $\equiv \frac{7c^2}{7(c-3)}$
 $\equiv \frac{c^2}{c-3}$

★★★★★
Seek the common denominator
Simplify:
 $\frac{c+7}{3} - \frac{2c+3}{6}$
 $\equiv \frac{3(c+7)}{(c+7)(2c+3)} - \frac{6(c+3)}{(c+7)(2c+3)}$
 $\equiv \frac{6c+9-6c-12}{(c+7)(2c+3)} \equiv \frac{-3}{(c+7)(2c+3)}$

★★★★★
 $\frac{12(2f+13) - 9(f+17)}{(f+17)(2f+13)} = 3$
 $\frac{24f+156-9f-153}{2f^2+13f+34f+221} = 3$
 $\frac{15f+3}{2f^2+47f+221} = 3$
 $15f+3 = 6f^2+141f+663$
 $6f^2+126f+660 = 0$ $(f+10)(f+11) = 0$
 $f^2+21f+110 = 0$ $f = -10$ $f = -11$

Change the subject

★
Make c the subject:
 $c + e = b$
 $-e \quad -e$
 $c = b - e$

★★
Make d the subject:
 $dc - f = b$
 $+f \quad +f$
 $dc = b + f$
 $\div c \quad \div c$
 $d = \frac{b+f}{c}$

★★★
Make c the subject:
 $\frac{cb - a}{e} = d \times e$
 $cb - a = de$
 $+a \quad +a$
 $cb = de + a$
 $\div b \quad \div b$
 $c = \frac{de+a}{b}$

★★★★
Make a the subject:
 $\frac{\sqrt{a-c}}{b} = e \times b$
 $\sqrt{a-c} = eb$
 $+c \quad +c$
 $\sqrt{a} = eb + c$
 $a = (eb+c)^2$

★★★★★
Make r the subject:
 $A = \pi r^2$
 $\div \pi \quad \div \pi$
 $\frac{A}{\pi} = r^2$
 $\sqrt{\frac{A}{\pi}} = r$

★★★★★
Make c the subject:
 $e = ac + 2c$ factorise
 $e = c(a+2)$
 $\frac{e}{a+2} = c$

Find $y = mx + c$ from a graph

★
What is the y-intercept?

Crosses at $(0, -4)$

★★
Calculate the gradient of the following line:

 $\frac{-2}{1} = -2$

★★★
Calculate the gradient of the following line:

 $\frac{2}{3}$

★★★★
Calculate the gradient of the following line:

 $\frac{-5}{3}$

★★★★★
What is the gradient?

 $\frac{2}{1} = 2$

★★★★★
State $y = mx + c$ for:

Crosses at $(0, 3)$ so $c = 3$
Gradient = $\frac{5-3}{1-0} = \frac{2}{1} = 2$
 $y = 2x + 3$

Rearranging into the form $y = mx + c$

I do, you do example

I DO

By rearranging the following into the form $y = mx + c$, what is the gradient value?

$$y - 2x - 2 = 0$$

Co-ordinates on the line

$$y = mx + c$$

Gradient y-intercept

Rearranging:

$$y - 2x - 2 = 0$$

$$y - 2x = 2$$

$$y = 2x + 2$$

Gradient is 2

YOU DO

By rearranging the following into the form $y = mx + c$, what is the gradient value?

$$y - 4x = -5$$

$$y = 4x - 5$$

Gradient is 4

Find the equation of a parallel line

I do, you do example

I DO

Write down the equation of a line parallel to $y = -5 + 8x$ that passes through the point $(-3, -26)$.

The equation of a straight line always comes in the form:

Co-ordinates on the line

$$y = mx + c$$

y-intercept

Coefficient of x is the gradient

Parallel lines have the same gradient. With $y = -5 + 8x$, 8 is the gradient.

$$y = 8x + c$$

Use the given coordinate to find c :

$$\begin{aligned} -26 &= 8x - 3 + c \\ -26 &= -24 + c \rightarrow c = -2 \\ y &= 8x - 2 \end{aligned}$$

YOU DO

Write down the equation of a line parallel to $y = -9x - 6$ that passes through the point $(6, -48)$.

Gradient of the parallel line

$$y = -9x + c$$

Use the coordinate to find c

$$\begin{aligned} -48 &= -9 \times 6 + c \\ -48 &= -54 + c \rightarrow c = 6 \\ y &= -9x + 6 \end{aligned}$$

Quadratic and cubic functions

I do, you do example

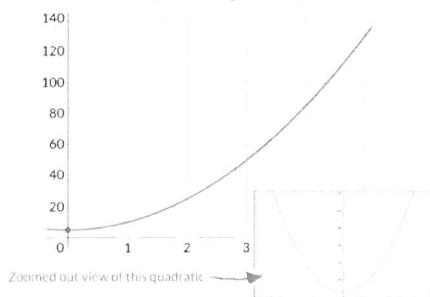
I DO

Draw the graph of $y = 5x^2 + 5$ for $0 \leq x \leq 5$.

To draw a graph, construct a table of values:

x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$5x^2$	0	5	20	45	80	125
y	5	10	25	50	85	130

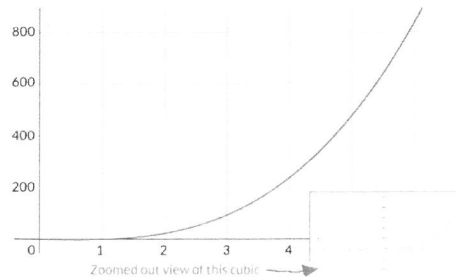
Now plot x against y :



YOU DO

Draw the graph of $y = 4x^3 - 5x$ for $1 \leq x \leq 6$.

x	1	2	3	4	5	6
x^3	1	8	27	64	125	216
$4x^3$	4	32	108	256	500	864
$5x$	5	10	15	20	25	30
y	-1	22	93	236	475	834



1★ Qu 1
What is the y-intercept?
 $y = 2x - 2$
 $(0, -2)$

3★ Qu 2
What is the gradient value?
 $y = 7x + 2$
7

3★ Qu 3
 $-7x + y = 2$
 $+7x \quad +7x$
 $y = 7x + 2$
 $(0, 2)$

3★ Qu 4
 $4x + y + 5 = 0$
 $-5 \quad -5$
 $4x + y = -5$
 $-4x \quad -4x$
 $y = -4x - 5$
-4

3★ Qu 5
 $6y + 8x = 3$
 $-8x \quad -8x$
 $6y = -8x + 3$
 $\div 6$
 $y = \left(-\frac{8}{6}\right)x + \frac{1}{2}$
 $-\frac{4}{3} = -1\frac{1}{3}$

3★ Qu 6
 $8y - 9x = 4$
 $+9x \quad +9x$
 $= 8y = 9x + 4$
 $\div 8$
 $y = \left(\frac{9}{8}\right)x + \frac{1}{2}$
 $\frac{9}{8} \rightarrow$

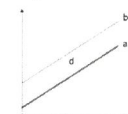
★
Line Q passes through (7,4) and (13,13). Line N passes through (-7,-10) and (-6,-8). Is line Q parallel to line N? Find the gradients.
Line Q gradient = $\frac{13-4}{13-7} = \frac{9}{6} = \frac{3}{2}$
Line N gradient = $\frac{-8-(-10)}{-6-(-7)} = \frac{2}{1} = 2$
 $\frac{3}{2} \neq 2$ so no they are not

★★
Write down the equation of a line parallel to $y = 4x - 6$.
Gradient of parallel line
Any line that follows $y = 4x + c$ is parallel to $y = 4x - 6$
e.g. $y = 4x + 2$

★★★
The equations of 2 lines are given below:
Gradient $y = 4x + 5$
 $y = 4 - 4x$ Gradient
Are they parallel?
Gradient of line 1 is 4 and the gradient of line 2 is -4, so no they are not parallel

Find the equation of a parallel line

★★★★
Write down the equation of a line parallel to $y = -5 + 2x$ that passes through the point (9,11).
 $y = 2x + c$
ii $= 2 \times 9 + c$
ii $= 18 + c \rightarrow c = -7$
 $y = 2x - 7$

★★★★★
If line a is $y = 5x + 5$ and d = 6 units, find the equation of the parallel line b.


Gradient of line a is 5. This means for every 1 on the x-axis, you go up 5 on the y-axis. If the horizontal difference is 6, $6 \times 5 = 30$ steps up. Each point on b is 30 higher than on a, inc. the y-intercept.
 $y = 5x + 35$

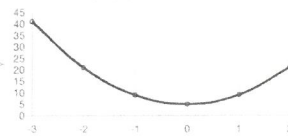
★
If $y = x^2$, complete the following table (rounding where appropriate):

x	-3	-2	-1	0
y	9	4	1	0

★★
If $y = x^3 - 5x^2 + 2$, complete the following table (rounding where appropriate):

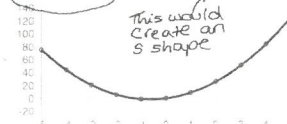
x	-1	0	1	2
y	-4	2	-2	-10

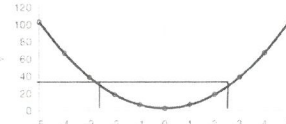
$\rightarrow x^3$ -1 0 1 8
 $\rightarrow x^2$ 1 0 1 4
 $\rightarrow 5x^2$ 5 0 5 20

★★★
Answer:


x	-3	-2	0	1	2
x^2	9	4	0	1	4
$4x^2$	36	16	0	4	16
y	41	21	5	9	21

Quadratic and cubic functions

★★★★
Which function is shown?
 $y = 4x^2 + 5x$ or $y = 3x^3 - 4x$
This would create an S shape


★★★★★
Use the graph to find x when $y = 34.4$.

 $x \approx -2.6$ $x \approx 2.6$

Solve linear equations (unknown both sides)

I do, you do example

Solve
 $7x + 3 = -74 - 4x$

The purpose of solving an equation is to find out the value of the unknown, x . A good first step is to get rid of x from one side of the equals sign:

$$7x + 3 = -74 - 4x$$

Remove the smaller number of x 's

Use inverse operations to eliminate terms from one side

$$11x + 3 = -74$$

$$11x = -77$$

$$x = 7$$

I DO

YOU DO

Solve

$$3 - 8x = -5 - 4x$$

$$3 = -5 + 4x$$

$$8 = 4x$$

$$2 = x$$

$$x = 2$$

Expand 2 or more binomials

I do, you do example

Expand

Method 1: The claw

Method 2: The grid

x	x	$+3$
x	x^2	$+3x$
-8	$-8x$	-24

$$x^2 + 3x - 8x - 24$$

Simplify

$$x^2 - 5x - 24$$

I DO

YOU DO

Expand

$$(x-2)(x+8)$$

$$x^2 + 8x - 2x - 16$$

Simplify

$$x^2 + 6x - 16$$

Factorise a quadratic expression

I do, you do example

Factorise

$$3x^2 + 10x + 8$$

To factorise, you want 2 numbers that multiply together to give ac but add together to give b .

$$ac = 3 \times 8 = 24 \quad b = 10$$

4×6 $4 + 6$

Rewrite the problem with b separated into $4x$ and $6x$:

Factorise each half

$$3x^2 + 6x + 4x + 8$$

Must be the same

$$3x(x+2) + 4(x+2)$$

Fully factorise

$$(3x+4)(x+2)$$

I DO

YOU DO

Factorise

$$2x^2 + 15x + 25$$

$$ac = 2 \times 25 = 50 \quad b = 15$$

5×10 $5 + 10$

$$2x^2 + 10x + 5x + 25$$

$$2x(x+5) + 5(x+5)$$

$$(2x+5)(x+5)$$

Solve linear equations (unknown both sides)

★
Solve
 $5x + 2 = 3x + 14$
 $-3x \quad -3x$
 $2x = 12$
 $\div 2 \quad \div 2$
 $x = 6$

★★
Solve
 $8x + 4 = -8 - 4x$
 $+4x \quad +4x$
 $12x + 4 = -8$
 $-4 \quad -4$
 $12x = -12$
 $\div 12 \quad \div 12$
 $x = -1$

★★★
Solve
 $10x + 6 = 2(2x - 18)$
 $10x + 6 = 4x - 36$
 $-4x \quad -4x$
 $6x + 6 = -36$
 $-6 \quad -6$
 $6x = -42$
 $\div 6 \quad \div 6$
 $x = -7$

★★★★
Solve
 $10x + 6 = 2(2x - 18)$
 $10x + 6 = 4x - 36$
 $-4x \quad -4x$
 $6x + 6 = -36$
 $-6 \quad -6$
 $6x = -42$
 $\div 6 \quad \div 6$
 $x = -7$

★★★★★
I think of a number, x times it by 6 and I add 2. The answer is 2 times my number add 38. What was my number?
 $6x + 2 = 2x + 38$
 $-2x \quad -2x$
 $4x + 2 = 38$
 $-2 \quad -2$
 $4x = 36$
 $\div 4 \quad \div 4$
 $x = 9$

★★★★★
Opposite sides of a rectangle are $(8x + 2)$ cm and $(26 - 4x)$ cm in length. Find the length of 1 side
 $8x + 2 = 26 - 4x$
 $+4x \quad +4x$
 $12x + 2 = 26$
 $-2 \quad -2$
 $12x = 24$
 $\div 12 \quad \div 12$
 $x = 2$
So 1 side is $8(2) + 2 = 16 + 2 = 18$

Expand 2 or more binomials

★
Expand
 $(x + 4)(x + 4)$
 $x^2 + 4x + 4x + 16$
 $x^2 + 8x + 16$

★★
Expand
 $(x - 2)(x - 3)$
 $x^2 - 3x - 2x + 6$
 $x^2 - 5x + 6$

★★★
Expand
 $(x + 4)(x - 4)$
 $x^2 - 4x + 4x - 16$
 $x^2 - 16$

★★★★
Expand
 $(4x + 8)(8x + 3)$
 $32x^2 + 12x + 64x + 24$
 $32x^2 + 76x + 24$

★★★★★
Expand
 $(3x^2 + 3)(5x^3 + 7)$
 $15x^5 + 21x^2 + 15x^3 + 21$
 $15x^5 + 15x^3 + 21x^2 + 21$

★★★★★
Expand
 $(x + 8)(x + 8)(x + 7)$
 $(x^2 + 8x + 8x + 64)(x + 7)$
 $(x^2 + 16x + 64)(x + 7)$
 $x^3 + 7x^2 + 16x^2 + 112x + 64x + 448$
 $x^3 + 23x^2 + 176x + 448$

Factorise a quadratic expression

★
Factorise
 $x^2 + 12x + 32$
 $ac = 1 \times 32 = 32 \quad b = 12$
 $4 \times 8 \quad 4 + 8$
 $x^2 + 4x + 8x + 32$
 $x(x + 4) + 8(x + 4)$
 $(x + 8)(x + 4)$

★★
Factorise
 $x^2 - 5x - 14$
 $ac = 1 \times -14 = -14 \quad b = -5$
 $-7 \times 2 \quad -7 + 2$
 $x^2 + 2x - 7x - 14$
 $x(x + 2) - 7(x + 2)$
 $(x - 7)(x + 2)$

★★★
Factorise
 $x^2 - 7x + 10$
 $ac = 1 \times 10 = 10 \quad b = -7$
 $-2 \times -5 \quad -2 + -5$
 $x^2 - 2x - 5x + 10$
 $x(x - 2) - 5(x - 2)$
 $(x - 5)(x - 2)$

★★★★
Factorise
 $x^2 - 36$
Rewrite the problem
 $x^2 + 0x - 36$
 $ac = 1 \times -36 = -36 \quad b = 0$
 $-6 \times 6 \quad -6 + 6$
 $x^2 - 6x + 6x - 36$
 $x(x - 6) + 6(x - 6)$
 $(x + 6)(x - 6)$

★★★★★
Factorise
 $3x^2 + 16x + 16$
 $ac = 3 \times 16 = 48 \quad b = 16$
 $4 \times 12 \quad 4 + 12$
 $3x^2 + 12x + 4x + 16$
 $3x(x + 4) + 4(x + 4)$
 $(3x + 4)(x + 4)$

★★★★★
Factorise
 $12x^2 + 14x + 4$
 $ac = 12 \times 4 = 48 \quad b = 14$
 $6 \times 8 \quad 6 + 8$
 $12x^2 + 6x + 8x + 4$
 $6x(2x + 1) + 4(2x + 1)$
 $(6x + 4)(2x + 1)$

Solve a quadratic by factorising

I do, you do example

Solve the following quadratic by factorising:

$$x^2 - 10x + 25 = 0$$

The quadratic is in the form $ax^2 + bx + c$

To factorise a quadratic, think of 2 numbers that multiply to give ac and add to give b :

$$ac = 1 \times 25 = 25 \quad b = -10$$

So -5 and -5 Split b in the quadratic into these 2 numbers

Factorise each side of the green line

$$x^2 - 5x - 5x + 25 = 0$$

$$x(x-5) - 5(x-5) = 0$$

$$(x-5)(x-5) = 0$$

Common bracket

If the product of the 2 brackets is 0, it means 1 or both must equal 0 too:

$$x-5=0 \Rightarrow x=5 \quad x-5=0 \Rightarrow x=5$$

I DO

YOU DO

Solve the following quadratic by factorising:

$$x^2 - 9x + 14 = 0$$

$$a=1 \quad b=-9 \quad c=14$$

$$ac = 1 \times 14 = 14 \quad b = -9$$

So -7 and -2

$$x^2 - 2x - 7x + 14 = 0$$

$$x(x-2) - 7(x-2) = 0$$

$$(x-7)(x-2) = 0$$

$$x-7=0 \quad x-2=0$$

$$\Rightarrow x=7 \quad \Rightarrow x=2$$

Complete the square

I do, you do example

Write the following expression in completed square form:

$$x^2 + 4x - 1$$

The completed square form is:

$$(x+d)^2 + e$$

$\frac{1}{2}b$ $c-d^2$

So for a quadratic above that is written in the form $ax^2 + bx + c$:

$$a = 1, b = 4 \text{ and } c = -1$$

$$d = \frac{1}{2}(4) = 2$$

$$e = -1 - (2)^2 = -1 - 4 = -5$$

$$(x+2)^2 - 5$$

From this form, we can also get the turning point of the quadratic: $(-2, -5)$

I DO

YOU DO

Write the following expression in completed square form:

$$x^2 - 6x - 2$$

$$a=1 \quad b=-6 \quad c=-2$$

$$d = \frac{1}{2}(-6) = -3$$

$$e = -2 - (-3)^2 = -2 - 9 = -11$$

$$(x-3)^2 - 11$$

Solve a quadratic by completing the square

I do, you do example

Solve the following quadratic by completing the square (leave in exact form):

$$x^2 + 2x - 2 = 0$$

Complete the square

$$(x+1)^2 - 2 - (1)^2 = 0$$

$$(x+1)^2 - 2 - 1 = 0$$

$$(x+1)^2 - 3 = 0$$

Begin to rearrange to make x the subject

$$(x+1)^2 = 3$$

$$x+1 = \pm\sqrt{3}$$

$$x = -1 \pm \sqrt{3}$$

There are 2 answers in this case due to the + and - of the root of 3

I DO

YOU DO

Solve the following quadratic by completing the square (leave in exact form):

$$x^2 - 4x - 4 = 0$$

$$(x-2)^2 - 4 - (-2)^2 = 0$$

$$(x-2)^2 - 4 - 4 = 0$$

$$(x-2)^2 - 8 = 0$$

$$(x-2)^2 = 8$$

$$x-2 = \pm\sqrt{8}$$

$$x = 2 \pm \sqrt{8}$$

Solve the following quadratic by factorising:

$$x^2 + 10x + 21 = 0$$

$$ac = 21 \quad b = 10$$

$$x^2 + 3x + 7x + 21 = 0$$

$$x(x+3) + 7(x+3) = 0$$

$$(x+7)(x+3) = 0$$

$$x+7=0 \quad x+3=0$$

$$x=-7 \quad x=-3$$

Solve the following quadratic by factorising:

$$x^2 - 3x - 18 = 0$$

$$ac = -18 \quad b = -3$$

$$x^2 - 6x + 3x - 18 = 0$$

$$x(x-6) + 3(x-6) = 0$$

$$(x+3)(x-6) = 0$$

$$x+3=0 \quad x-6=0$$

$$x=-3 \quad x=6$$

Solve the following quadratic by factorising:

$$x^2 - 9x + 20 = 0$$

$$ac = 20 \quad b = -9$$

$$x^2 - 5x - 4x + 20 = 0$$

$$x(x-5) - 4(x-5) = 0$$

$$(x-4)(x-5) = 0$$

$$x-4=0 \quad x-5=0$$

$$x=4 \quad x=5$$

Solve the following quadratic by factorising:

$$x^2 - 64 = 0$$

$$ac = -64 \quad b = 0$$

$$x^2 + 8x - 8x - 64 = 0$$

$$x(x+8) - 8(x+8) = 0$$

$$(x-8)(x+8) = 0$$

$$x-8=0 \quad x+8=0$$

$$x=8 \quad x=-8$$

Solve the following quadratic by factorising:

$$2x^2 - 13x + 15 = 0$$

$$ac = 30 \quad b = -13$$

$$2x^2 - 10x - 3x + 15 = 0$$

$$2x(x-5) - 3(x-5) = 0$$

$$(2x-3)(x-5) = 0$$

$$2x-3=0 \quad x-5=0$$

$$2x=3 \Rightarrow x=1.5 \quad x=5$$

Solve the following quadratic by factorising:

$$10x^2 + 23x + 12 = 0$$

$$ac = 120 \quad b = 23$$

$$10x^2 + 15x + 8x + 12 = 0$$

$$5x(2x+3) + 4(2x+3) = 0$$

$$(5x+4)(2x+3) = 0$$

$$5x+4=0 \quad 2x+3=0$$

$$5x=-4 \quad 2x=-3$$

$$x=-0.8 \quad x=-1.5$$

Complete the square

Find the value of a in the following:

$$x^2 + 8x + 19 = (x+4)^2 + a$$

$$a=1 \quad b=8 \quad c=19$$

$$e = 19 - 4^2$$

$$= 19 - 16 = 3$$

$$a = 3$$

Find the value of a in the following:

$$x^2 - 18x + 88 = (x-9)^2 + a$$

$$a=1 \quad b=-18 \quad c=88$$

$$e = 88 - (-9)^2$$

$$= 88 - 81 = 7$$

$$a = 7$$

Write the following expression in completed square form:

$$x^2 - 2x - 7$$

$$a=1 \quad b=-2 \quad c=-7$$

$$d = \frac{1}{2}(-2) = -1$$

$$e = -7 - (-1)^2$$

$$= -7 - 1 = -8$$

$$(x-1)^2 - 8$$

Write the following expression in completed square form:

$$x^2 + 7x + 7$$

$$a=1 \quad b=7 \quad c=7$$

$$d = \frac{1}{2}(7) = 3.5$$

$$e = 7 - (3.5)^2$$

$$= 7 - 12.25 = -5.25$$

$$(x+3.5)^2 - 5.25$$

Find the minimum point of the following function:

$$f(x) = x^2 - 6x - 3$$

$$a=1 \quad b=-6 \quad c=-3$$

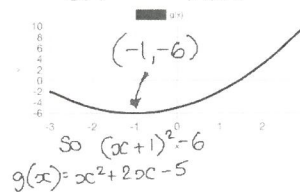
$$d = \frac{1}{2}(-6) = -3$$

$$e = -3 - (-3)^2 = -3 - 9 = -12$$

$$(x-3)^2 - 12$$

Turning point at (3, -12)

f(x) = x^2 has translated to g(x). Identify g(x).



Solve a quadratic by completing the square

Solve the following quadratic by completing the square (leave in exact form):

$$x^2 + 6x - 8 = 0$$

$$(x+3)^2 + a = 0$$

$$(x+3)^2 - 17 = 0$$

$$(x+3)^2 = 17$$

$$x+3 = \pm\sqrt{17}$$

$$x = -3 \pm\sqrt{17}$$

Solve the following quadratic by completing the square (leave in exact form):

$$x^2 - 4x - 1 = 0$$

$$(x-2)^2 + a = 0$$

$$(x-2)^2 - 5 = 0$$

$$(x-2)^2 = 5$$

$$x-2 = \pm\sqrt{5}$$

$$x = 2 \pm\sqrt{5}$$

Solve the following quadratic by completing the square (leave in exact form):

$$x^2 - 2x - 4 = 0$$

$$(x-1)^2 - 5 = 0$$

$$(x-1)^2 = 5$$

$$x-1 = \pm\sqrt{5}$$

$$x = 1 \pm\sqrt{5}$$

$$x^2 - 3x - 5 = 0$$

$$(x-1.5)^2 - 7.25 = 0$$

$$(x-1.5)^2 = 7.25$$

$$x-1.5 = \pm\sqrt{7.25}$$

$$x = 1.5 \pm\sqrt{7.25}$$

$$5(x^2 - x) - 3 = 0$$

$$5(x-0.5)^2 - 0.25 - 3 = 0$$

$$5(x-0.5)^2 - 3.25 = 0$$

$$5(x-0.5)^2 - 4.25 = 0$$

$$5(x-0.5)^2 = 4.25$$

$$(x-0.5)^2 = 0.85$$

$$x-0.5 = \pm\sqrt{0.85}$$

$$x = 0.5 \pm\sqrt{0.85}$$

$$x = 1.42 \quad x = -0.42$$

$$8(x^2 + \frac{17}{8}x) - 18 = 0$$

$$8((x + \frac{17}{16})^2 - (\frac{17}{16})^2) - 18 = 0$$

$$8((x + \frac{17}{16})^2 - \frac{289}{16}) - 18 = 0$$

$$8(x + \frac{17}{16})^2 - \frac{289}{2} - 18 = 0$$

$$8(x + \frac{17}{16})^2 - 27\frac{1}{2} = 0$$

$$8(x + \frac{17}{16})^2 = 27\frac{1}{2} \Rightarrow x = \frac{17}{16} \pm \sqrt{\frac{216}{4}}$$

$$x + \frac{17}{16} = \pm\sqrt{216\frac{1}{4}} \Rightarrow x = -2.90$$

Iterations

I do, you do example

Given $x_0 = 3$,
find x_3 to 2dp:

$$x_{n+1} = \sqrt[3]{\frac{7x_n^2 + 9x_n + 10}{3}}$$

With iteration, you use the output from a calculation as the new input of the next one.

$$x_1 = \sqrt[3]{\frac{7 \times 3^2 + 9 \times 3 + 10}{3}} = \sqrt[3]{\frac{7 \times 9 + 27 + 10}{3}} = \sqrt[3]{\frac{63 + 27 + 10}{3}} = \sqrt[3]{\frac{100}{3}}$$

$x_1 = \text{Ans} = 3.21829794$

$$x_2 = \sqrt[3]{\frac{7 \times \text{Ans}^2 + 9 \times \text{Ans} + 10}{3}} = 3.3368861$$

$$x_3 = 3.40 \text{ to 2dp}$$

Use "Ans" on your calculator to bring in the answer to the previous calculation to save time. Click "=" to keep repeating the calculation.

I DO

YOU DO

Given $x_0 = 1$,
find x_3 to 2dp:

$$x_{n+1} = \sqrt[3]{\frac{6x_n^2 + 7x_n + 6}{10}}$$

$$x_1 = \sqrt[3]{\frac{6 \times 1^2 + 7 \times 1 + 6}{10}} = \sqrt[3]{\frac{6 + 7 + 6}{10}} = \sqrt[3]{\frac{19}{10}}$$

$$x_1 = 1.23856233$$

$$x_2 = 1.336521687$$

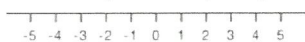
$$x_3 = 1.376361505$$

$$x_3 = 1.38 \text{ to 2dp}$$

Read/Represent inequalities on a number line

I do, you do example

Identify the inequality:



$$x > 2$$

Any value greater than 2 satisfies the inequality

When representing an inequality on a number line, use:

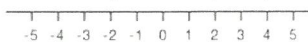
- hollow dot for $<$ and $>$
- solid dot for \leq and \geq

The arrow indicates the direction of the size of the numbers that satisfy the inequality.

I DO

YOU DO

Identify the inequality:



Any number less than or equal to 2 satisfies the inequality
 $x \leq 2$

Understand geometric sequences

I do, you do example

Given that the following is a geometric sequence, write the next 2 terms:
18, 54, 162, ...

Unlike an arithmetic sequence where you have a common difference between two terms, a geometric sequence has a common ratio.

Common ratio = $\frac{54}{18} = 3$ ← To find the next term, multiply the previous term by 3

$$162 \times 3 = 486 \quad 486 \times 3 = 1458$$

The rule for this sequence could be represented by an iterative process:

$$u_1 = 18 \quad u_{n+1} = 3u_n$$

Next term → ← Previous term

I DO

YOU DO

Given that the following is a geometric sequence, write the next 2 terms:
6, 12, 24, ...

$$\frac{12}{6} = 2 \quad \leftarrow \text{Doubles each time}$$

$$24 \times 2 = 48$$

$$48 \times 2 = 96$$

★

If $a_1 = 2$ and $a_{n+1} = 3a_n - 4$, find a_2 .

$$a_2 = 3 \times a_1 - 4$$

$$a_2 = 3 \times 2 - 4$$

$$= 6 - 4 = 2$$

★★

If $b_1 = 2$ and $b_{n+1} = 4b_n - 3$, find b_3 .

$$b_2 = 4 \times b_1 - 3$$

$$= 4 \times 2 - 3$$

$$= 8 - 3 = 5$$

$$b_3 = 4 \times 5 - 3$$

$$= 20 - 3 = 17$$

★★★

The population, P , of Algebra town can be modelled by $P_{n+1} = 1.7(P_n + 40)$. In 2021, $P = 1000$. What is P in 2022?

$$P_{2021} = 1000$$

$$P_{2022} = 1.7(1000 + 40)$$

$$P_{2022} = 1.7 \times 1040$$

$$= 1768$$

★★★★

Given $x_0 = 1$, find x_3 to 2dp:

$$x_{n+1} = \sqrt[3]{\frac{7x_n^2 - 7x_n + 8}{3}}$$

$$x_1 = 1.3867225$$

$$x_2 = 1.5764761$$

$$x_3 = 1.69 \text{ to 2dp}$$

★★★★★

Make this x the subject

$$11x^3 - 2x^2 - 6x - 10 = 0$$

$$11x^3 = 2x^2 + 6x + 10$$

$$x^3 = \frac{2x^2 + 6x + 10}{11}$$

$$x_{n+1} = \sqrt[3]{\frac{2x_n^2 + 6x_n + 10}{11}}$$

★★★★★

$$2x^3 - 9x^2 - 5x - 7 = 0$$

has a solution between 5 and 6. Find x to 2dp using:

$$x_{n+1} = \sqrt[3]{\frac{9x_n^2 + 5x_n + 7}{2}}$$

$x_0 = 5$	$x_4 = 2.87027$
$x_1 = 3.0050506$	$x_5 = 2.8681$
$x_2 = 2.9057892$	$x_6 = 2.8674$
$x_3 = 2.87809596$	$x = 2.87$

★

Identify the inequality:

A number less than 3

$$x < 3$$

★★

Identify the inequality:

A number greater than or equal to -1

$$x \geq -1$$

★★★

Identify the inequality:

A number greater than 1

$$x > 1$$

Read/Represent inequalities on a number line

★★★★

Answer:

★★★★★

Identify the inequality:

A number greater than 0 but less than or equal to 3

$$0 < x \leq 3$$

★★★★★

Answer:

★

Is it a geometric sequence?
2, 5, 7, 12, ...

Ratio between term 1 and 2

$$\frac{5}{2} = 2.5$$

Ratio between term 2 and 3

$$\frac{7}{5} = 1.4$$

No common ratio so it is not geometric

★★

Given that the following is a geometric sequence, write the next 2 terms:
10, 20, 40, ...

$$\frac{20}{10} = 2 \text{ (Doubles)}$$

$$40 \times 2 = 80$$

$$80 \times 2 = 160$$

★★★

Given that the following is a geometric sequence, write the next 2 terms:
3, 0.6, 0.12, ...

$$\frac{0.6}{3} = 0.2 \text{ (Next term is a fifth of the last)}$$

$$0.12 \times 0.2 = 0.024$$

$$0.024 \times 0.2 = 0.0048$$

Understand geometric sequences

★★★★

A geometric sequence has a first term of 3 and a common ratio of 0.2. Find the first 3 terms.

$$3 \times 0.2 = 0.6$$

$$0.6 \times 0.2 = 0.12$$

$$3, 0.6, 0.12$$

★★★★★

If $u_1 = 8$, use the following to find u_3 :

$$u_{n+1} = 0.3u_n$$

$$u_2 = 0.3 \times 8 = 2.4$$

$$u_3 = 0.3 \times 2.4 = 0.72$$

★★★★★

Using the following rule, if $u_2 = 12$, find u_5 :

$$u_{n+1} = 3u_n$$

$$u_3 = 3 \times 12 = 36$$

$$u_4 = 3 \times 36 = 108$$

$$u_5 = 3 \times 108 = 324$$

I do, you do example

Here are the first 3 terms of an arithmetic sequence:

$$c, c + 2h, c + 4h$$

Find the nth term rule.

To find the nth term, we can use the general rule:

$$a n + b$$

Common difference 0th term

$$c-2h \quad c \quad c+2h \quad c+4h$$

0th term Common difference

$$\begin{aligned} n^{\text{th}} \text{ term} &= (2h)n + c - 2h \\ &= 2hn - 2h + c \\ &= 2h(n-1) + c \end{aligned}$$

Here are the first 3 terms of an arithmetic sequence:

$$e, e + 2a, e + 4a$$

Find the nth term rule.

$$e-2a \quad e \quad e+2a \quad e+4a$$

$$\begin{aligned} n^{\text{th}} \text{ term} &= (2a)n + e - 2a \\ &= 2an - 2a + e \\ &= 2a(n-1) + e \end{aligned}$$

Solve linear simultaneous equations

I do, you do example

Find the value of a and b:

Label the equations

$$\begin{aligned} \textcircled{1} \quad &5a + 3b = 32 \\ \textcircled{2} \quad &4a + 2b = 24 \end{aligned}$$

Take multiples of the two equations (if required) to get either the coefficient of a or b the same

$$\begin{aligned} \textcircled{1} \times 2 \quad \textcircled{3} \quad &10a + 6b = 64 \\ \textcircled{2} \times 3 \quad \textcircled{4} \quad &12a + 6b = 72 \end{aligned}$$

Eliminate b

$$\begin{aligned} \textcircled{4} - \textcircled{3} \quad &2a = 8 \\ &a = 4 \end{aligned}$$

If a = 4 use an equation to now find b. Substitute a into the equation:

$$\begin{aligned} \textcircled{1} \quad &5 \times 4 + 3b = 32 \\ &20 + 3b = 32 \\ &3b = 12 \\ &b = 4 \end{aligned}$$

Find the value of a and b:

$$\textcircled{1} \quad 5a + 2b = 16$$

$$\textcircled{2} \quad 2a - 4b = -8$$

$$\begin{aligned} \textcircled{1} \times 2 \quad \textcircled{3} \quad &10a + 4b = 32 \\ \textcircled{2} \quad &2a - 4b = -8 \end{aligned}$$

$$\begin{aligned} \textcircled{3} + \textcircled{2} \quad &12a = 24 \\ &a = 2 \end{aligned}$$

Substitute a = 2 into $\textcircled{1}$

$$\begin{aligned} 5 \times 2 + 2b &= 16 \\ 10 + 2b &= 16 \\ 2b &= 6 \\ b &= 3 \end{aligned}$$

Form expressions and equations

I do, you do example

I think of a number (n), I subtract 3 and then multiply it by 6. Write an expression, in terms of n, to represent my number.

Statements like these always provide order that we need to preserve when considering BIDMAS.

$$\begin{aligned} \textcircled{1} \quad &\text{I think of a number} \quad n \\ \textcircled{2} \quad &\text{I subtract 3} \quad n-3 \\ \textcircled{3} \quad &\text{multiply by 6} \quad 6(n-3) \end{aligned}$$

(n - 3) is placed in brackets to ensure the subtraction happens before the multiplication.

I think of a number (n), I add 6 and then multiply it by 3. Write an expression, in terms of n, to represent my number.

$$\begin{aligned} \text{I think of a number} \quad &n \\ \text{I add 6} \quad &n+6 \\ \text{I multiply by 3} \quad &3(n+6) \end{aligned}$$

★
Here are the first 3 terms of an arithmetic sequence:
 $3h, 3h + g, 3h + 2g$
Find the next 2 terms.
 $3h, 3h+g, 3h+2g$
 $4^{th} \text{ term} = 3h + 2g + g = 3h + 3g$
 $5^{th} \text{ term} = 3h + 3g + g = 3h + 4g$

★★
Here are the first 3 terms of a geometric sequence:
 e, ea, ea^2
Find the next 2 terms.
 e, ea, ea^2
 $4^{th} \text{ term} = ea^2 \times a = ea^3$
 $5^{th} \text{ term} = ea^3 \times a = ea^4$

★★★
Here are the first 3 terms of an arithmetic sequence:
 $2d, 2d + g, 2d + 2g$
Show that the 6th term is:
 $2d + 5g$
 $4^{th} \text{ term} = 2d + 2g + g = 2d + 3g$
 $5^{th} \text{ term} = 2d + 3g + g = 2d + 4g$
 $6^{th} \text{ term} = 2d + 4g + g = 2d + 5g$

★★★★
Here are the first 3 terms of an arithmetic sequence:
 $3d, 3d + g, 3d + 2g$
Find the nth term rule.
 $n^{th} \text{ term} = g \cdot n + 3d - g$
 $= gn - g + 3d$
 $= g(n-1) + 3d$

★★★★★
Here are the first 3 terms of an arithmetic sequence:
 $2c, 2c + 2e, 2c + 4e$
If the third term is 10 and the sixth term is 28, find c and e.
 $6^{th} \text{ term} = 2c + 10e$
① $2c + 10e = 28$
② $2c + 4e = 10$
① - ② $6e = 18$
 $e = 3$
Subs $e = 3$ into ②
 $2c + 12 = 10$
 $2c = -2$
 $c = -1$

★
Find the value of a and b:
① $a + 7b = 50$
② $a + 6b = 44$
① - ② $b = 6$
Substitute $b = 6$ into ②
 $a + 6 \times 6 = 44$
 $a + 36 = 44$
 $a = 8$

★★
Find the value of a and b:
① $5a + b = 22$
② $8a - b = 17$
① + ② $13a = 39$
 $a = 3$
Substitute $a = 3$ into ①
 $5 \times 3 + b = 22$
 $15 + b = 22$
 $b = 7$

★★★
Find the value of a and b:
① $3a + 5b = 32$
② $2a + 4b = 24$
① $\times 2$ ③ $6a + 10b = 64$
② $\times 3$ ④ $6a + 12b = 72$
④ - ③ $2b = 8$
 $b = 4$
Substitute $b = 4$ into ①
 $3a + 5 \times 4 = 32$
 $3a + 20 = 32$
 $3a = 12$
 $a = 4$

★★★★
① $4c + 2f = 14$
② $5c + 2f = 16$
② - ① $c = 2$ ← Cost of a pack of crisps
Substitute $c = 2$ into ①
 $4 \times 2 + 2f = 14$
 $8 + 2f = 14$ ← Cost of a pack of fish
 $2f = 6$
 $f = 3$

★★★★★
Find the value of a and b:
① $5a + 2b = 14$ ② $4a - 3b = 2$
 $4a = 2 + 3b$
① $\times 3$ ③ $15a + 6b = 42$
② $\times 2$ ④ $8a - 6b = 4$
③ + ④ $23a = 46 \rightarrow a = 2$
 $10 + 2b = 14$
 $2b = 4 \rightarrow b = 2$

★★★★★
If $a = 2^p, b = 2^q, ab = 16$ and $2ab^3 = 512$, find the value of p and q.
 $2^p \times 2^q = 16 \rightarrow 2^{p+q} = 2^4$ ← Laws of Indices
 $2^p + q = 4$
 $2^p \times 2^q \times 2^{3q} = 512 \rightarrow 2^{p+4q} = 2^9$
 $p + 4q = 9$
② - ① $1 + 2q = 5$
 $2q = 4$
 $q = 2$
Subs $q = 2$ into ①
 $p + 2 = 4$
 $p = 2$

Solve linear simultaneous equations

★
Ashton is 5 years older than Liam. If Ashton is y years old, how old is Liam?
 $y - 5$


★★
I think of a number (n), I multiply it by 3 and add 2. Write an expression, in terms of n, to represent my number.
 $3n + 2$

★★★
I think of a number (n), I add 2 and then divide it by 4. Write an expression, in terms of n, to represent my number.
 $\frac{(n+2)}{4}$
or $(n+2) \div 4$

Form expressions and equations

★★★★
Steve is y years old. Matt is 2 years older than Steve. Sue is 6 years younger than Steve. Find the sum of their ages.
 $y + (y+2) + (y-6) = 3y - 4$

★★★★★
I think of a number (n), I multiply it by 9 and then add 7. If the answer is my original number subtract 17. What is my number?
 $9n + 7 = n - 17$
 $8n + 7 = -17$
 $8n = -24$
 $n = -3$

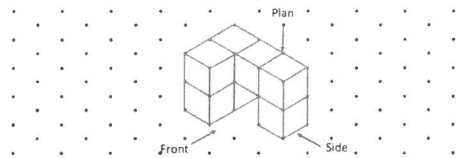
★★★★★
If $x = n + 6, y = n - 8$ and $z = 3n + 4$, write a formula for the perimeter, P, in terms of n.

 $(n+6) + (n-8) + (3n+4) = 5n + 2$

Plans and elevations

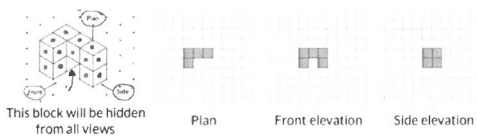
I do, you do example

I DO

Draw the plan, front and side elevation for the following shape:

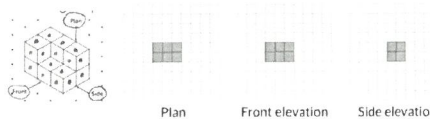
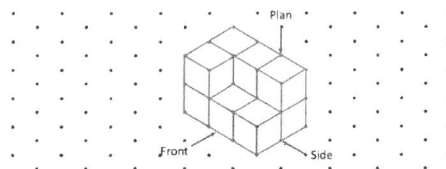


A plan view is a drawing of the shape from above (bird's eye view). It is often a good idea to colour the sides that you would see from each perspective to help draw the different views.



YOU DO

Draw the plan, front and side elevation for the following shape:



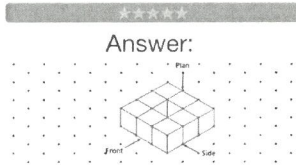
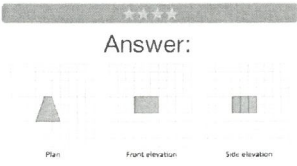
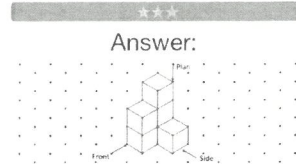
Angles on parallel lines

Fact sheet

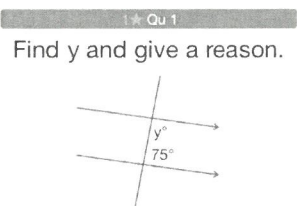
Angles in polygons

Fact sheet

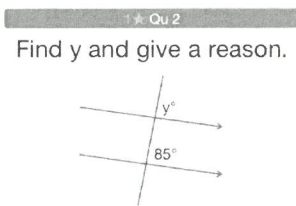
★
Answer:
6



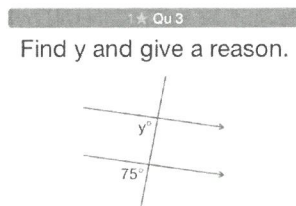
★★★★★
Answer:
Square based pyramid



$y = 105^\circ$ since y and 75 are co-interior angles

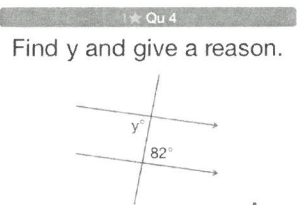


$y = 85^\circ$ since y and 85 are corresponding angles

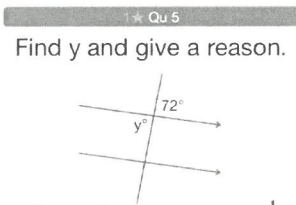


$y = 75^\circ$ since y and 75 are corresponding angles

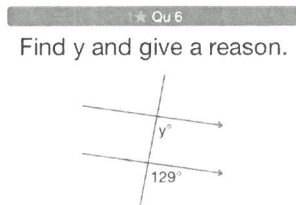
Angles on parallel lines



$y = 82^\circ$ since y and 82 are alternate angles



$y = 72^\circ$ since y and 72 are vertically opposite angles



$y = 129^\circ$ since y and 129 are corresponding angles

★
Find the interior angle sum of a 13-sided polygon.
Using $(n-2) \times 180$
 $(13-2) \times 180$
 $= 11 \times 180$
 $= 1980^\circ$

★★
7 of the angles in a 8-sided polygon are $118^\circ, 131^\circ, 124^\circ, 133^\circ, 134^\circ, 130^\circ$ and 132° . Calculate the size of the last angle.
Using $(n-2) \times 180$
 $(8-2) \times 180 = 6 \times 180$
Sum of all angles $\rightarrow = 1080$
 $1080 - (118 + 131 + 124 + 133 + 134 + 130 + 132) = 178^\circ$

★★★
Will a regular 8-sided polygon tessellate? Give a reason for your answer.
 $(8-2) \times 180 = 1080$
Sum of all angles
 $1080 \div 8 = 135^\circ$
1 angle
No it will not tessellate since an angle in the polygon is not a factor of 360.

Angles in polygons

★★★★
How many sides does a regular n-sided polygon have if 1 exterior angle is 21.2° (rounded to 1 decimal place)?
Using the fact all exterior angles sum to 360° :
 $360 \div 21.2 \approx 17$ sides

★★★★★
The interior angle of a regular n-sided polygon 128.6° (rounded to 1 decimal place). Calculate the value of n.
 $180 - 128.6 = 51.4$ ← 1 exterior angle
 $360 \div 51.4 \approx 7$ sides

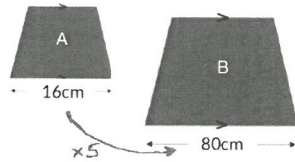
★★★★★
 $(5-2) \times 180 = 540^\circ$ ← Sum of all angles in a pentagon
 $540 \div 5 = 108^\circ$ ← 1 interior angle in the pentagon
 $108 - 60 = 48^\circ$ ← Marked angle
 $(180 - 48) \div 2 = 66^\circ$
Same length making it isosceles

Area/Volume scale factors

I do, you do example

I DO

A and B are similar shapes. The area of A is 80cm^2 . Find the area of B?



Similar shapes are 2 of the same shape but different in size.

Their lengths are linked by a scale factor.
Their areas are linked by $(\text{scale factor})^2$.
Their volumes are linked by $(\text{scale factor})^3$.

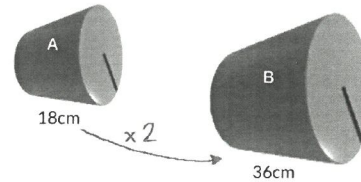
$$\text{Scale factor} = \frac{80}{16} = 5$$

$$\text{Area scale factor} = 5^2 = 25$$

$$\text{So area of shape B} = 80 \times 25 = 2000\text{cm}^2$$

YOU DO

A and B are similar. The volume of B is 720cm^3 . Find the volume of A?



$$\text{Scale factor} = \frac{36}{18} = 2$$

$$\text{Volume scale factor} = 2^3 = 8$$

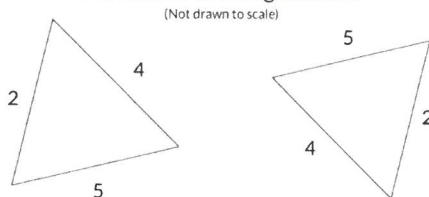
$$\text{So volume of shape A} = 720 \div 8 = 90\text{cm}^3$$

Congruence

I do, you do example

I DO

The two triangles are congruent. State the condition of congruence.



Congruent means one shape can become another by rotating, reflecting or translating it.

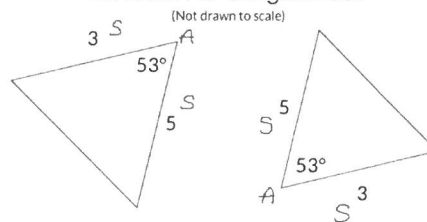
Two triangles are congruent if:

- SSS - 3 sides are equal
- ASA - 2 angles & a corresponding side are equal
- AAS - 2 angles & a corresponding side are equal
- SAS - 2 sides & the angle between them are equal
- RHS - A right angle, hypotenuse & a corresponding side are equal

Here SSS is satisfied

YOU DO

The two triangles are congruent. State the condition of congruence.



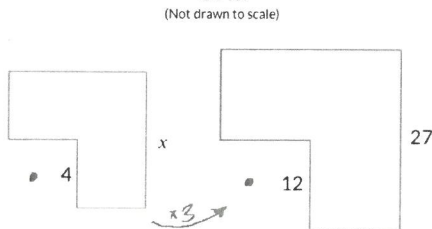
In this case they are congruent because each have SAS that are the same $5, 53^\circ, 5$

Similar shapes

I do, you do example

I DO

The two shapes are similar. Find the value of x .



Two shapes are said to be similar if one shape can become another after resizing (and rotating, reflecting or translating)

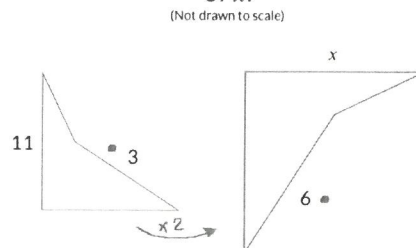
The lengths of the sides in one shape will have a scale factor of the same corresponding side in the other.

$$\text{Scale factor of corresponding sides} = \frac{12}{4} = 3$$

$$\text{So } x \times 3 = 27, x = 9$$

YOU DO

The two shapes are similar. Find the value of x .

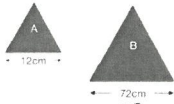


$$\text{Scale factor of corresponding sides} = \frac{6}{3} = 2$$

$$\text{So } 11 \times 2 = x$$

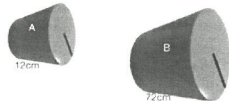
$$x = 22$$

★
A and B are similar shapes.
The area of A is 36cm^2 .
Find the area of B?



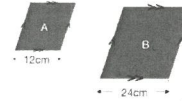
Scale factor = $\frac{24}{12} = 2$
Area scale factor = $2^2 = 4$
Area of B = $36 \times 4 = 144\text{cm}^2$

★★
A and B are similar. The volume of A is 60cm^3 . Find the volume of B?



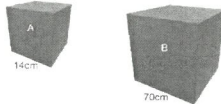
Scale factor = $\frac{24}{12} = 2$
Volume scale factor = $2^3 = 8$
Volume of B = $60 \times 8 = 480\text{cm}^3$

★★★
A and B are similar shapes. The area of B is 192cm^2 . Find the area of A?

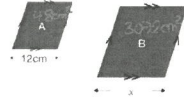


Scale factor = $\frac{24}{12} = 2$
Area scale factor = $2^2 = 4$
Area of A = $192 \div 4 = 48\text{cm}^2$

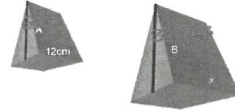
★★★★
Scale factor = $\frac{70}{14} = 5$
Volume scale factor = $5^3 = 125$
Volume of A = $\frac{8750}{125} = 70\text{cm}^3$



★★★★★
Area scale factor = $\frac{392}{48} = 8$
Scale factor = $\sqrt{8} = 2.828$
 $x = 12 \times 2.828 = 33.94\text{cm}$

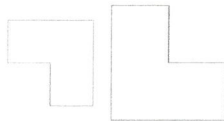


★★★★★
Volume scale factor = $\frac{3840}{60} = 64$
Scale factor = $\sqrt[3]{64} = 4$
 $x = 12 \times 4 = 48\text{cm}$



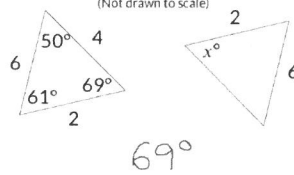
Congruence

★
Are the two shapes congruent?



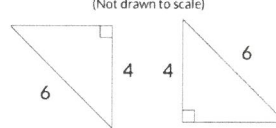
No, one is an enlargement of the other so they are similar not congruent

★★
The two triangles are congruent. What is x?



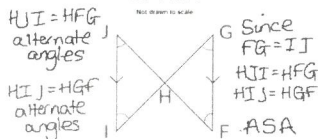
69°

★★★
The two triangles are congruent. State the condition of congruence.



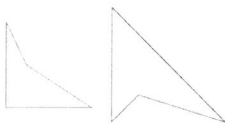
Right angle
Hypotenuse
Side

★★★★
If $FG = IJ$, prove that triangle FGH is congruent to HIJ .



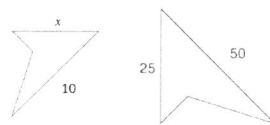
Similar shapes

★
Are the two shapes similar?



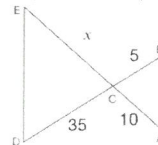
No since they are different shapes.

★★
The two shapes are similar. Find the value of x.



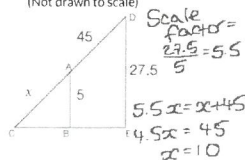
Scale factor = $\frac{50}{10} = 5$
 $x \times 5 = 25$ so $x = \frac{25}{5} = 5$

★★★
Triangle ABC is similar to CDE. Find the value of x.

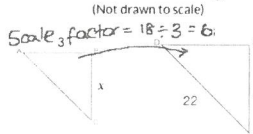


Scale factor = $\frac{35}{5} = 7$
so $CD = 7BC$
and $CE = 7AC$
 $= 7 \times 10 = 70$

★★★★
If $AB = 5$, $DE = 27.5$ and $AD = 45$, what is AC?



★★★★★
Triangle ABC is similar to DEF. Find the value of x to the nearest integer.



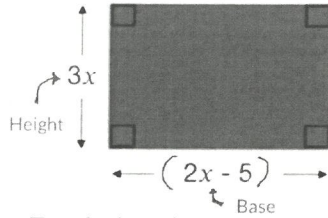
Use Pythagoras to find this length $\sqrt{22^2 - 18^2} = \sqrt{160}$
So $x = \frac{\sqrt{160}}{6} = 2.08185$
So $x \approx 2$

Algebraic area of 2D shapes

I do, you do example

I DO

Write an expression to represent the area of the following shape:



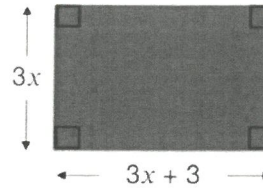
To calculate the area of a rectangle, multiply the base by the perpendicular height:

$$\begin{aligned} \text{Area} &= (2x - 5) \times 3x \\ &= 2x \times 3x - 5 \times 3x \\ &= (6x^2 - 15x) \text{ cm}^2 \end{aligned}$$

Expand

YOU DO

Write an expression to represent the area of the following shape:



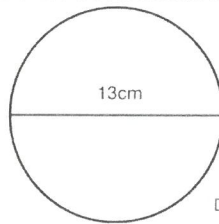
$$\begin{aligned} \text{Area} &= (3x + 3) \times 3x \\ &= 3x \times 3x + 3 \times 3x \\ &= (9x^2 + 9x) \text{ cm}^2 \end{aligned}$$

Circumference of a circle

I do, you do example

I DO

Calculate the circumference to 1dp.



Distance from 1 side of a circle to the other through the middle

Use the formula:

$$\text{Circumference} = \pi \times \text{diameter}$$

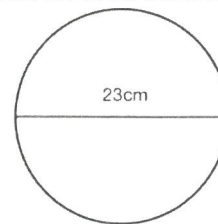
Perimeter of a circle Find it on your calculator

$$\begin{aligned} C &= \pi \times 13 \\ C &= 13\pi \text{ cm} \\ C &= 40.8 \text{ cm} \end{aligned}$$

In terms of π *To 1 d.p.*

YOU DO

Calculate the circumference to 1dp.



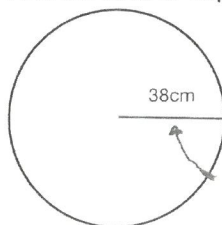
$$\begin{aligned} C &= \pi \times d \\ C &= \pi \times 23 \\ C &= 23\pi \text{ cm} \\ C &= 72.3 \text{ cm} \end{aligned}$$

Area of a circle

I do, you do example

I DO

Find the area to 1dp.



Distance from the centre of a circle to the circumference

Use the formula:

$$\text{Area of a circle} = \pi \times \text{radius}^2$$

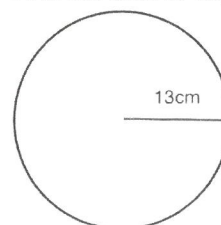
Find it on your calculator Square the radius

$$\begin{aligned} A &= \pi \times 38^2 \\ A &= 1444\pi \text{ cm}^2 \\ A &= 4536.5 \text{ cm}^2 \end{aligned}$$

Answer in terms of π *To 1 d.p.*

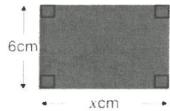
YOU DO

Find the area to 1dp.



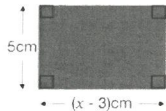
$$\begin{aligned} A &= \pi \times r^2 \\ A &= \pi \times 13^2 \\ A &= 169\pi \text{ cm}^2 \\ A &= 530.9 \text{ cm}^2 \end{aligned}$$

Write an expression to represent the area of the following shape.



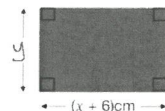
$$\text{Area} = x \times 6 = 6x \text{ cm}^2$$

Write an expression to represent the area of the following shape.



$$\begin{aligned} \text{Area} &= (x-3) \times 5 \\ &= x \times 5 - 3 \times 5 \\ &= (5x-15) \text{ cm}^2 \end{aligned}$$

The area of the following shape is $(6x + 36) \text{ cm}^2$. What is the length of the missing side.



Factorise $6x + 36$
 $6(x+6)$
 $y = 6$

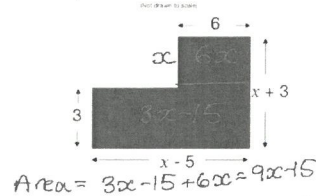
Write an expression to represent the area of the following shape.

$$\begin{aligned} \text{Area} &= (4x-8) \times 6x \\ &= 4x \times 6x - 8 \times 6x \\ &= (24x^2 - 48x) \text{ cm}^2 \end{aligned}$$

Calculate the shaded area.

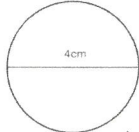
$$\begin{aligned} \text{Area}_A &= (x-2) \times 4 = 4x-8 \\ \text{Area}_B &= (x-7) \times 2 = 2x-14 \\ \text{Area} &= (4x-8) - (2x-14) \\ &= (2x+6) \text{ cm}^2 \end{aligned}$$

Write an expression to represent the area of:



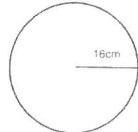
$$\text{Area} = 3(x-5) + 6(x+3) = 9x+15$$

Calculate the circumference to 1dp.



$$\begin{aligned} C &= \pi \times d \\ C &= \pi \times 4 \\ C &= 4\pi \text{ cm} \\ C &= 12.6 \text{ cm (to 1dp)} \end{aligned}$$

Calculate the circumference to 1dp.

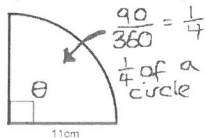


$$\begin{aligned} \text{Diameter} &= 2 \times \text{radius} \\ &= 2 \times 16 \\ &= 32 \text{ cm} \\ C &= \pi \times d \\ C &= \pi \times 32 \\ C &= 32\pi \text{ cm} \\ C &= 100.5 \text{ cm (to 1dp)} \end{aligned}$$

If the circumference of a circle is 47.1cm to 1dp, calculate the diameter of the circle to 1dp.

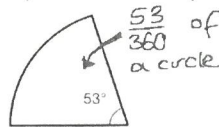
$$\begin{aligned} C &= \pi \times d \\ d &= C \div \pi \\ d &= 47.1 \div \pi \\ d &= 15.0 \text{ cm (to 1dp)} \end{aligned}$$

Find the perimeter to 1dp.



$$\begin{aligned} \frac{90}{360} &= \frac{1}{4} \\ \frac{1}{4} \text{ of a circle} \\ P &= \frac{\theta}{360} \times \pi \times d + 2r \\ P &= \frac{1}{4} \times \pi \times 22 + 2 \times 11 \\ P &= 39.3 \text{ cm (to 1dp)} \end{aligned}$$

Find the perimeter to 1dp.



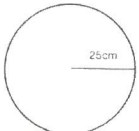
$$\begin{aligned} \frac{53}{360} \text{ of a circle} \\ P &= \frac{\theta}{360} \times \pi \times d + 2r \\ P &= \frac{53}{360} \times \pi \times 62 + 2 \times 31 \\ P &= 90.7 \text{ cm (to 1dp)} \end{aligned}$$

If the perimeter is 38.5cm to 1 dp, calculate theta to 1dp.



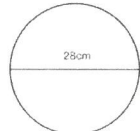
$$\begin{aligned} P &= \frac{\theta}{360} \times \pi \times d + 2r \\ 38.5 &= \frac{\theta}{360} \times \pi \times 28 + 2 \times 14 \\ \frac{10.5 \times 360}{28\pi} &= \theta = 43.0^\circ \text{ (to 1dp)} \end{aligned}$$

Find the area to 1dp.



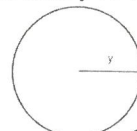
$$\begin{aligned} A &= \pi \times r^2 \\ A &= \pi \times 25^2 \\ A &= 625\pi \text{ cm}^2 \\ A &= 1963.5 \text{ cm}^2 \text{ (to 1dp)} \end{aligned}$$

Find the area to 1dp.



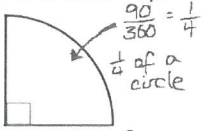
$$\begin{aligned} \text{Radius} &= \frac{1}{2} \text{ of diameter} \\ &= 28 \div 2 \\ &= 14 \text{ cm} \\ A &= \pi \times r^2 \\ A &= \pi \times 14^2 \\ A &= 196\pi \text{ cm}^2 \\ A &= 615.8 \text{ cm}^2 \text{ (to 1dp)} \end{aligned}$$

If the area is 153.9cm², calculate y to 1dp.



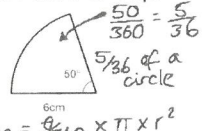
$$\begin{aligned} A &= \pi \times r^2 \\ r^2 &= A \div \pi \\ r &= \sqrt{A \div \pi} \\ r &= \sqrt{153.9 \div \pi} \\ r &= 7.0 \text{ cm (to 1dp)} \end{aligned}$$

Find the area to 1dp.



$$\begin{aligned} \frac{90}{360} &= \frac{1}{4} \\ \frac{1}{4} \text{ of a circle} \\ \text{Area} &= \frac{\theta}{360} \times \pi \times r^2 \\ &= \frac{1}{4} \times \pi \times 10^2 \\ &= 25\pi \text{ cm}^2 = 78.5 \text{ cm}^2 \text{ (to 1dp)} \end{aligned}$$

Find the area to 1dp.



$$\begin{aligned} \frac{50}{360} \text{ of a circle} \\ \text{Area} &= \frac{\theta}{360} \times \pi \times r^2 \\ &= \frac{50}{360} \times \pi \times 6^2 \\ &= 5\pi \text{ cm}^2 = 15.7 \text{ cm}^2 \text{ (to 1dp)} \end{aligned}$$

If the area is 37.5cm² to 1 dp, calculate theta to 1dp.



$$\begin{aligned} \text{Area} &= \frac{\theta}{360} \times \pi \times r^2 \\ \theta &= \frac{\text{Area} \times 360}{\pi \times r^2} \\ &= \frac{37.5 \times 360}{\pi \times 9^2} \\ &= 53.1^\circ \text{ (to 1dp)} \end{aligned}$$

Apply SOH CAH TOA

I do, you do example

I DO

Find y to 1dp.

Opposite to the angle → Opp 9cm

Adjacent is the side next to the angle → Adj ycm

Hypotenuse is the side opposite to the right angle

33° θ

Greek symbol used to denote an angle

Choose from:

$$\sin \theta = \frac{\text{Opp}}{\text{Hyp}} \quad \cos \theta = \frac{\text{Adj}}{\text{Hyp}} \quad \tan \theta = \frac{\text{Opp}}{\text{Adj}}$$

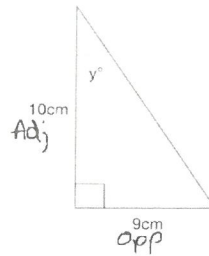
Use this since we have opp and need adj

$$\tan(33) = \frac{9}{y} \quad y = \frac{9}{\tan(33)} \approx 13.9\text{cm}$$

Rearrange

YOU DO

Find y to 1dp.



$$\tan \theta = \frac{\text{Opp}}{\text{Adj}}$$

Use tan since we have both opp & adj

$$\tan y = \frac{9}{10} = 0.9$$

$$y = \tan^{-1}(0.9)$$

$$y \approx 42.0^\circ$$

Trigonometry exact values

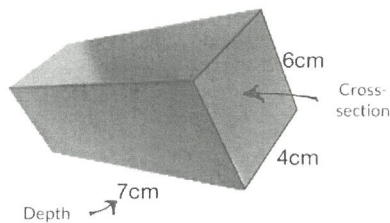
Fact sheet

Volume of 3D shapes

I do, you do example

I DO

Calculate the volume of the cuboid:



To calculate the volume of a prism or 3D shape where the cross-section is the same throughout, use:

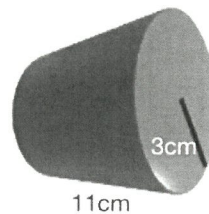
Volume = Area of cross-section x Depth

$$\text{Volume} = (6 \times 4) \times 7 = 168\text{cm}^3$$

Note the units of volume

YOU DO

Calculate the volume to 1dp.



$$\text{Volume} = \pi r^2 \times \text{depth}$$

$$= \pi \times 3^2 \times 11$$

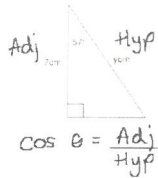
$$= \pi \times 9 \times 11$$

Answer in terms of π

$$= 99\pi \text{ cm}^3$$

$$= 311.0 \text{ cm}^3$$

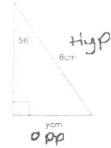
Using $\cos 57^\circ \approx 0.5$, find y to 1dp.



$$\cos \theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\cos 57^\circ = \frac{7}{y} \quad \text{Rearrange} = 14.0 \text{ cm}$$

Find y to 1dp.

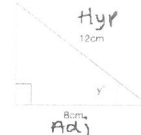


$$\sin \theta = \frac{\text{Opp}}{\text{Hyp}}$$

$$\sin 56^\circ = \frac{6}{y} \quad y = \sin(56) \times 6$$

$$y \approx 5.0 \text{ cm}$$

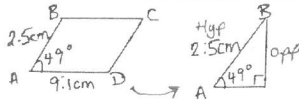
Find y to 1dp.



$$\cos \theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\cos y = \frac{8}{12} = \frac{2}{3} \quad y = \cos^{-1}(\frac{2}{3})$$

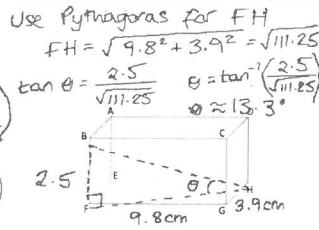
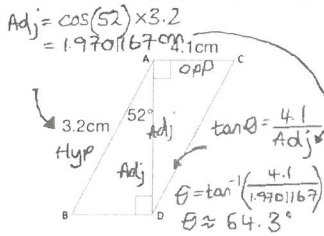
$$y \approx 48.2^\circ$$



$$\sin \theta = \frac{\text{Opp}}{\text{Hyp}}$$

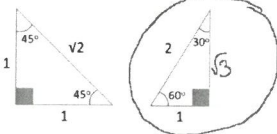
$$\sin 49^\circ = \frac{\text{Opp}}{2.5} \quad \text{Opp} = \sin(49) \times 2.5$$

$$\approx 1.9 \text{ cm}$$



Trigonometry exact values

Using the triangles, find the exact value of $\tan 60^\circ$.



$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Find the exact value of: $\cos 45^\circ$

Using the isosceles right angled triangle

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

Find the exact value of:

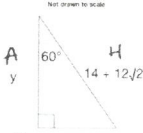
$$\sin 45^\circ + \cos 60^\circ$$

$$\frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$= \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}} = \frac{2 + \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{2\sqrt{2} + 2}{4} = \frac{\sqrt{2} + 1}{2}$$

Find the exact value of y .



$$\cos 60^\circ = \frac{y}{14 + 12\sqrt{2}}$$

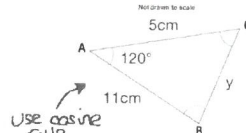
$$\frac{1}{2} = \frac{y}{14 + 12\sqrt{2}} \quad y = 7 + 6\sqrt{2}$$

Find the exact value of: $\sin 120^\circ$

Same answer as $\sin 60^\circ$

$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

Find the exact value of y .



Use cosine rule

$$y^2 = 11^2 + 5^2 - 2 \times 11 \times 5 \times \cos 120^\circ$$

$$y^2 = 121 + 25 - 110 \cos 120^\circ$$

$$y^2 = 201 \quad y = \sqrt{201}$$

Volume of 3D shapes

What is the correct mathematical name for the following shape?



Hexagonal prism

$$\text{Volume} = \pi \times r^2 \times \text{depth}$$

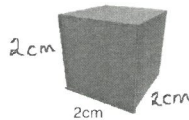
$$= \pi \times 2^2 \times 11$$

$$= \pi \times 4 \times 11$$

$$= 44\pi \text{ cm}^3$$

$$= 138.2 \text{ cm}^3$$

Calculate the volume of the following cube:



$$\text{Volume} = (2 \times 2) \times 2$$

$$= 8 \text{ cm}^3$$

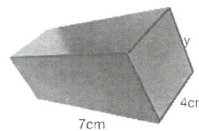
$$\text{Volume} = (\frac{1}{2} \times b \times h) \times \text{depth}$$

$$= (\frac{1}{2} \times 2 \times y) \times 10 = 50$$

$$10y = 50$$

$$y = 5 \text{ cm}$$

The cuboid has a volume of 56 cm^3 . Calculate y .



$$\text{Volume} = (y \times 4) \times 7 = 56$$

$$28y = 56$$

$$y = 2 \text{ cm}$$